Thermoelastic Properties of a Novel Fuzzy Fiber-Reinforced Composite

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The effective thermoelastic properties of a fuzzy fiber-reinforced composite (FFRC) have been estimated by employing the generalized method of cells approach and the Mori–Tanaka method. The novel constructional feature of this fuzzy fiber-reinforced composite is that the uniformly aligned carbon nanotubes (CNTs) are radially grown on the circumferential surface of the horizontal carbon fibers. Effective thermoelastic properties of the fuzzy fiber-reinforced composite estimated by the generalized method of cells approach have been compared with those predicted by the Mori–Tanaka method. The present work concludes that the axial thermal expansion coefficient of the fuzzy fiber-reinforced composite slightly increases for the lower values of the carbon fiber volume fraction, whereas the transverse thermal expansion coefficient of the fuzzy fiber-reinforced composite significantly decreases over those of the composite without CNTs. Also, the results demonstrate that the effect of temperature variation on the effective thermal expansion coefficients of the fuzzy fiber-reinforced composite is negligible. [DOI: 10.1115/1.4023691]

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1 Introduction

The discovery of CNTs [1] has stimulated extensive research on the prediction of their mechanical, electrical, and thermal properties. Ruoff and Lorents [2] examined some aspects of the mechanical and thermal properties of multiwalled carbon nanotubes and single-walled carbon nanotubes in terms of the known elastic properties of graphite. Many research investigations revealed that the axial Young’s modulus of CNTs is in terapascal range [3–8]. Batra and Sears [9] proposed that the axis of transverse isotropy of a CNT is a radial line rather than the centroidal axis of the CNT and found that the Young’s modulus in the radial direction equals about one fourth of that in the axial direction. Batra and Gupta [10] determined the wall thickness and material moduli of a CNT based on the frequencies of axial, torsional, and radial breathing modes. An atomistic-based continuum model has been developed by Cheng et al. [11] for the estimation of the mechanical properties of single-walled carbon nanotubes. Tsai et al. [12] modeled the hollow cylindrical molecular structure of CNTs as an equivalent transversely isotropic solid cylinder and employed the molecular mechanics approach to determine the elastic properties of CNTs. In order to exploit the exceptional elastic properties of CNTs, extensive research is being carried out for developing the CNT-reinforced composites [12–20]. A great deal of research has also been carried out on the potential applications of CNTs, such as nanoelectronics [21–24]. The CNT-based nanoelectronic devices may experience temperature variation that leads to thermal deformations and residual stresses in the devices. Therefore, the coefficient of thermal expansion (CTE) of CNTs is an important property for CNT-based nanoelectronics. The X-ray diffraction technique has been employed by several researchers to estimate the CTEs of CNTs and their bundles [25–27]. Jiang et al. [28] developed an analytical method to determine the CTEs of single-walled carbon nanotubes and reported that the CTEs are negative at low and room temperature and become positive at high temperature. This is similar to the experiments of Yosida [26] and Maniwa et al. [27], which exhibited strong temperature dependence of CTEs of CNT bundles. Performing the molecular dynamics simulations, Kwon et al. [29] reported that the axial and radial CTEs of CNTs are nonlinear functions of change in temperature.

Although a great prospect has been highlighted through the above-mentioned research on CNT-reinforced composites, the manufacturing of such unidirectional continuous CNT-reinforced composites in large scale has to encounter some challenging difficulties. Typical among these are the agglomeration of CNTs, the misalignment, and the difficulty in manufacturing very long CNTs. Also, the weak cohesive force between carbon fibers and the matrix causes low shear and bending strengths. Fiber coating is often used to modify the carbon fiber interfacial characteristics and improve the strength of the composite. The hybridization of carbon fibers with CNTs is a new way of surface modification. CNTs on the carbon fiber/matrix interface improve the interfacial shear strength and out-of-plane stiffness properties of the fiber-reinforced composites. Traditional fiber-reinforced composites have excellent in-plane properties but have poor out-of-plane properties. Further research on improving the out-of-plane properties of the composites and the better use of CNTs led to the growth of CNTs on the surfaces of the advanced fibers. For example, Bower et al. [30] have grown aligned CNTs on the substrate surface using high-frequency microwave plasma-enhanced chemical vapor deposition with an extreme-case example of conformal radial growth of CNTs on the surface of a hair-thin optical fiber glass. They have found that the growth rate of CNTs is approximately 100 nm/s with entire growth of 12 μm, and such CNT growth always occur perpendicular to the substrate, regardless of the substrate shape. Qian et al. [31] experimentally investigated that the enhancement in the fiber/matrix interfacial shear strength can be obtained by growing CNTs on the surfaces of the fibers. Zhao et al. [32] synthesized different morphologies of multiwalled CNTs on the surfaces of carbon fibers by using the floating catalyst method. Veedu et al. [33] demonstrated that the remarkable improvements in the interlaminar fracture toughness, hardness, delamination resistance, in-plane mechanical properties, damping, and thermoelastic behavior of laminated composite can be
obtained by growing multiwalled CNTs about 60 μm long on the surfaces of the fibers. Their test results suggest that the presence of CNTs in the transverse (i.e., thickness) direction of composites reduces the effective CTE up to 62% compared to that of the base composite. Qiu et al. [34] presented a method to fabricate multifunctional multiscale composites through an effective infiltration-based vacuum-assisted resin transfer molding process. Their study indicated that the CTE of functionalized nanocomposites reduced up to 25.2% compared with that of the composite without multiwalled CNTs. Mathur et al. [35] experimentally investigated that the flexural strength and the modulus of the carbon-fiber-reinforced composite can be improved by growing CNTs on the surfaces of the carbon fibers. Garcia et al. [36] fabricated a hybrid laminate in which the reinforcements are a woven cloth of alumina fibers with in situ grown CNTs on the surfaces of the fibers. They demonstrated that electromechanical properties of such a laminate are enhanced because of the CNTs grown on the surfaces of the alumina fibers. Zhang et al. [37] investigated the effects of the growth of CNTs on the thermal, electrical, and mechanical properties of the fiber/polymer interface of a composite. Ray et al. [38] carried out a load transfer analysis of a short carbon fiber-reinforced composite, in which the aligned CNTs are radially grown on the surfaces of the carbon fibers. They have found that, if the carbon fiber is coated with radially aligned CNTs, then the axial load transferred to the fiber is reduced. Xiao et al. [39] examined advanced implementations of guided growth of CNTs on quartz substrates with different orientations. Their study revealed that angle-dependent van der Waals interactions between CNTs can account for all aspects of their alignment on quartz substrates. Recently, Ray [40] proposed a novel hybrid smart composite, which exhibits improved electromechanical properties because of the radial growing of CNTs on the piezoelectric fibers. Subsequently, Ray [41] developed a shear lag model of this novel smart composite to analyze the load transferred to the coated piezoelectric fibers from the CNT-reinforced matrix in the absence and the presence of the electric field. Most recently, a novel fuzzy fiber-reinforced composite (FFRC) reinforced with single-walled carbon nanotubes and carbon fibers is independently proposed by Kundalwal and Ray [42] and Chatzigeorgiou et al. [43]. The distinct constructional feature of such a fuzzy fiber-reinforced composite is that the uniformly aligned CNTs are radially grown on the circumferential surfaces of the unidirectional carbon fibers. Kundalwal and Ray [42] predicted that the transverse effective properties of this novel fuzzy fiber-reinforced composite are significantly enhanced over their values without CNTs.

Since the temperature variation may affect the thermoelastic properties of the CNT-reinforced composite, it is imperative to investigate the effect of temperature variation on the effective thermoelastic properties of the fuzzy fiber-reinforced composite. However, the thermoelastic properties of such nanocomposites being composed of fuzzy fiber reinforcements have not yet been studied. In this paper, authors intend to estimate the thermoelastic properties of the fuzzy fiber-reinforced composite and investigate the effect of temperature variation on the effective thermoelastic properties of the fuzzy fiber-reinforced composite.  

## 2 Effective Thermoelastic Properties of the Fuzzy Fiber-Reinforced Composite

Figure 1 shows a schematic sketch of a lamina of the fuzzy fiber-reinforced composite. The constructional feature of such a continuous unidirectional carbon fiber-reinforced composite is that CNTs of equal length are uniformly spaced and radially grown on the surfaces of the carbon fiber reinforcements. CNTs considered here are transversely isotropic [7,12]. They are grown on the surfaces of the carbon fibers in such a way that their axes of transverse isotropy are normal to the surface of the fiber. Such a resulting fuzzy fiber is shown in Fig. 2. When this fuzzy fiber is embedded into the polymer material, the gap between two adjacent CNTs is filled with the polymer. Consequently, the radially aligned CNTs reinforce the polymer matrix surrounding the carbon fiber along the direction transverse to the length of the carbon fiber. Thus, the augmented fuzzy fiber can be viewed as a circular cylindrical composite fuzzy fiber (CFF), in which a carbon fiber is embedded in the CNT-reinforced polymer matrix nanocomposite (PMNC) and the diameter of the composite fuzzy fiber equals the sum of the diameter of the carbon fiber and the length of a CNT. Such a composite fuzzy fiber is schematically demonstrated in Fig. 3. Therefore, the representative volume element (RVE) of the proposed fuzzy fiber-reinforced composite can be treated as being composed of two phases, wherein the reinforcement is the composite fuzzy fiber and the matrix is the monolithic polymer material. Thus, the analytical procedure for estimating the effective thermoelastic properties of the fuzzy fiber-reinforced composite starts with the estimation of the effective thermoelastic properties of the polymer matrix nanocomposite material a priori. Subsequently, considering the polymer matrix nanocomposite material as the matrix phase and the carbon fiber as the reinforcement, effective thermoelastic properties of the composite fuzzy fiber are to be computed. Finally, using the thermoelastic properties of the composite fuzzy fiber and the monolithic polymer matrix, the effective thermoelastic properties of the proposed fuzzy fiber-reinforced composite can be estimated. The schematic diagram shown in Fig. 4 illustrates the various steps involved in the micromechanical modeling of the fuzzy fiber-reinforced composite. Also, composite fuzzy fibers are assumed to be uniformly...
an interphase is considered between a CNT and between carbon–carbon bonds \[46\]. In several research studies actions, which are obviously weaker than covalent bonding polymer takes place through van der Waals noncovalent inter-
naturally, the bonding between a CNT and its surrounding carbon atoms of a CNT and a surrounding polymer matrix.

2.1 Generalized Method of Cells (MOC) Approach. This section presents the micromechanics model based on the generalized method of cells approach \[44,45\] to estimate the effective thermoelastic properties of the polymer matrix nanocomposite material surrounding the carbon fiber, the composite fuzzy fiber, and the fuzzy fiber-reinforced composite, respectively.

2.1.1 Effective Thermoelastic Properties of the Polymer Matrix Nanocomposite (PMNC). This section presents the micromechanics model based on the generalized method of cells approach to estimate the effective thermoelastic properties of the polymer matrix nanocomposite material, which are required as inputs for computing the effective properties of the composite fuzzy fiber. In order to estimate the effective thermoelastic properties of the polymer matrix nanocomposite material surrounding the carbon fiber, the consideration of the nonbonded van der Waals interaction between an atom of CNT and an atom of the polymer matrix. Hence, in the present study, the CNT/polymer matrix interphase is not considered.

From the constructional feature of the composite fuzzy fiber, it may be viewed that the carbon fiber is wrapped by a lamina of the polymer matrix nanocomposite material. Such an unwound lamina of the polymer matrix nanocomposite is reinforced by CNTs along its thickness direction (i.e., along the 3-direction), shown in Fig. 5. The average effective thermoelastic properties of the polymer matrix nanocomposite material surrounding the carbon fiber may be approximated by estimating the effective thermoelastic properties of this unwound lamina. Assuming that CNTs are equivalent solid fibers \[12,15,18,20\] uniformly spaced in the polymer matrix and aligned along the 3-axis, the polymer matrix nanocomposite can be viewed to be composed of cells forming doubly periodic arrays along the 1– and the 2–directions. Such an arrangement of cells consisting of subcells has been illustrated in Fig. 6. Each rectangular parallelepiped subcell is labeled by \(\beta\gamma\), with \(\beta\) and \(\gamma\) denoting the location of the subcell along the 1– and the 2–directions, respectively. The numbers of subcells present in the fuzzy fiber-reinforced composite are estimated with and without the consideration of an interphase between a CNT and the polymer matrix by Kundalwal and Ray \[42\]. In their study, the consideration of the CNT/polymer matrix interphase does not affect the in-plane effective elastic coefficients, and its effects on the other effective elastic coefficients are also not pronounced. Thus, for conservative and intuitive estimates, one may neglect the nonbonded van der Waals interaction at an atom of CNT and the polymer matrix. Hence, in the present study, the CNT/polymer matrix interphase is not considered.
the cell along the 1– and 2–directions are M and N, respectively. Here, each cell represents the RVE and the subcell can be either a CNT or the matrix. Modeling the perfectly bonding condition at the interface between the subcells is the basis for deriving the micromechanics model using the generalized method of cells. It may be noted that such perfectly bonding conditions between the subcells can be established by satisfying the compatibility of displacements and continuities of tractions at the interfaces between the subcells of the cell. The volume \( V_{c} \) of each subcell is

\[
V_{c} = l_{b}b_{c}h_{c}
\]  

(1)

where \( b_{c} \), \( h_{c} \), and \( l_{b} \) denote the width, the height, and the length of the subcell, respectively, while the volume (\( V \)) of the cell is

\[
V = lh
\]  

(2)

Based on the principal material coordinate (1–2–3) axes, the constitutive relations for the medium of a subcell under thermal environment are given by

\[
\{ \sigma^{c} \} = [C^{c}]{\{ b^{c} \} - \{ x^{c} \}}\Delta T
\]  

(3)

where the stress vector, the strain vector, the thermal expansion coefficient vector, and the elastic coefficient matrix of the subcell represented by the superscript \( c \) are

\[
\{ \sigma^{c} \} = \{ \sigma^{c}_{1} \} \sigma^{c}_{2} \sigma^{c}_{3} \sigma^{c}_{12} \sigma^{c}_{13} \sigma^{c}_{23}
\]

\[
\{ b^{c} \} = \{ b^{c}_{1} \} \quad \{ b^{c}_{2} \} \quad \{ b^{c}_{3} \} \quad \{ b^{c}_{12} \} \quad \{ b^{c}_{13} \} \quad \{ b^{c}_{23} \}
\]

\[
\{ x^{c} \} = \{ x^{c}_{1} \} \quad \{ x^{c}_{2} \} \quad \{ x^{c}_{3} \} \quad 0 \quad 0 \quad 0
\]

(4)

Also, in Eq. (3), \( \Delta T \) is the temperature deviation from a reference temperature. It should be noted that, in order to utilize the constituent material properties during computation, the superscript \( c \) denoting the location of the subcell in the cell should be replaced by \( m \) or \( p \) as the medium of the subcell is CNT or polymer, respectively. In the generalized method of cells approach, the effective properties are determined by evaluating the properties of the repeating cells filled up with the equivalent homogeneous materials. This amounts to volume averaging of the field quantity in concern. Thus, the volume-averaged strains and stresses in the unwound polymer matrix nanocomposite can be expressed, respectively, as

\[
\{ \epsilon_{nc} \} = \frac{1}{V} \sum_{\beta=1}^{M} \sum_{\gamma=1}^{N} V_{\beta \gamma} \{ \epsilon^{\beta \gamma} \}
\]

\[
\{ \sigma_{nc} \} = \frac{1}{V} \sum_{\beta=1}^{M} \sum_{\gamma=1}^{N} V_{\beta \gamma} \{ \sigma^{\beta \gamma} \}
\]  

(5)

Here, the superscript \( nc \) designates the unwound polymer matrix nanocomposite. Imposition of the interfacial displacement continuities provides the following \( 2(M+N)+MN+1 \) number of relations between the volume-averaged subcell strains and the unwound polymer matrix nanocomposite strains

\[
\sum_{\beta=1}^{M} b_{p \beta} \epsilon^{\beta \gamma}_{1} = b_{e}^{\gamma_{c}}, \quad \gamma = 1, 2, \ldots, N
\]  

(6)

(7)

\[
\sum_{\gamma=1}^{N} h_{\beta} \epsilon^{\beta \gamma}_{2} = h_{e}^{\gamma_{c}}, \quad \beta = 1, 2, \ldots, M
\]

\[
\epsilon^{\beta \gamma}_{3} = \epsilon^{\beta \gamma}_{nc}, \quad \beta = 1, 2, \ldots, M, \quad \gamma = 1, 2, \ldots, N
\]  

(8)

(9)

\[
\sum_{\beta=1}^{M} \sum_{\gamma=1}^{N} b_{p \beta} h_{\gamma} \epsilon^{\beta \gamma}_{12} = b_{h}^{\gamma_{c}}, \quad \beta = 1, 2, \ldots, M
\]

\[
\sum_{\gamma=1}^{N} h_{\beta} \epsilon^{\beta \gamma}_{23} = h_{e}^{\gamma_{c}}, \quad \beta = 1, 2, \ldots, M
\]  

(10)

(11)

Imposition of the interfacial traction continuity conditions between the adjacent subcells yields the following \( 5MN - 2(M+N) - 1 \) number of relations between the volume-averaged subcell stresses

\[
\sigma^{\beta \gamma}_{1} = \sigma^{(\beta+1) \gamma}_{1}, \quad \beta = 1, 2, \ldots, M - 1, \quad \gamma = 1, 2, \ldots, N
\]  

(12)

\[
\sigma^{\beta \gamma}_{2} = \sigma^{(\beta+1) \gamma}_{2}, \quad \beta = 1, 2, \ldots, M, \quad \gamma = 1, 2, \ldots, N - 1
\]  

(13)

\[
\sigma^{\beta \gamma}_{3} = \sigma^{(\beta+1) \gamma}_{3}, \quad \beta = 1, 2, \ldots, M, \quad \gamma = 1, 2, \ldots, N - 1
\]  

(14)

\[
\sigma^{\beta \gamma}_{12} = \sigma^{(\beta+1) \gamma}_{12}, \quad \beta = 1, 2, \ldots, M - 1, \quad \gamma = 1, 2, \ldots, N - 1
\]  

(15)

\[
\sigma^{\beta \gamma}_{23} = \sigma^{(\beta+1) \gamma}_{23}, \quad \beta = 1, 2, \ldots, M, \quad \gamma = 1, 2, \ldots, N - 1
\]  

(16)

\[
\sigma^{\beta \gamma}_{31} = \sigma^{(\beta+1) \gamma}_{31}, \quad \beta = 1, \quad \gamma = 1, 2, \ldots, N - 1
\]  

(17)

Equations (6–11) form a set of \( 2(M+N)+MN+1 \) number of relations and can be arranged in a matrix form as follows:

\[
[A]_{G} \{ \epsilon_{s} \} = [B] \{ \epsilon^{nc} \}
\]  

(18)

in which \( \{ \epsilon_{s} \} \) is the \((6MN \times 1)\) vector of subcell strains assembled together and \( \{ \epsilon^{nc} \} \) is the \((6 \times 1)\) vector of composite strains, while \( [A]_{G} \) is the \([2(M+N)+MN+1] \times (6MN)\) matrix formed by the geometrical parameters of the subcells and the \( [B] \) matrix is constructed by the geometrical parameters of the cell.

Using the constitutive equations given by Eq. (3), the traction continuity equations \( 5MN - 2(M+N) - 1 \) can be expressed in the following matrix form:

\[
[A]_{G} \{ \epsilon_{s} \} - [A]_{G} \{ \epsilon_{s} \} = [D] \{ \epsilon_{s} \} \Delta T
\]  

(19)

in which \( \{ \epsilon_{s} \} \) is the \((6MN \times 1)\) thermal expansion coefficient vector of subcells assembled together and \( [A]_{G} \) is a \([5MN - 2(M+N) - 1] \times (6MN)\) matrix containing the elastic properties of the subcells. Combination of Eqs. (18) and (19) leads to

\[
[A]_{G} \{ \epsilon_{s} \} = [K] \{ \epsilon^{nc} \} + [D] \{ \epsilon_{s} \} \Delta T
\]  

(20)

where

\[
[A] = [A]_{G}, \quad [K] = \left[ \begin{array}{c} 0 \end{array} \right], \quad \text{and} \quad [D] = [A]_{G}
\]  

with \( \bar{0} \) and \( \bar{\epsilon} \) being \([5MN - 2(M+N) - 1] \times (6)]\) and \([2(M+N)+MN+1] \times (6MN)\) null vectors, respectively.
From Eq. (20), the subcell strains can be expressed in terms of the composite strains, as follows:

$$\{e_{sk}\} = \{A_{sk}\}\{e_{sc}\} + \{D_{sk}\}\{z_{sk}\}\Delta T$$  \hspace{1cm} (21)

where \(\{A_{sk}\} = [A]^{-1}[K]\) and \(\{D_{sk}\} = [A]^{-1}[D]\). The matrices \([A_{sk}\) and \([D_{sk}\) can be treated as the mechanical concentration matrix and thermal concentration matrix, respectively. It is now possible to extract the matrices \([A_{sc}^p]\) and \([D_{sc}^p]\) of strain concentration factors for each subcell from the matrices \([A_{sk}\) and \([D_{sk}\), respectively, such that each subcell strain can be expressed in terms of the composite strains and the thermal strains as follows:

$$\{e_{sp}\} = [A_{sp}^p]\{e_{sc}\} + [D_{sp}^p]\{z_{sc}\}\Delta T$$  \hspace{1cm} (22)

Substituting Eq. (22) into Eq. (3) yields

$$\{\sigma_{sp}\} = [C_{sp}]\{e_{sc}\} + \{\tau_{sp}\}$$  \hspace{1cm} (23)

Using Eq. (23) in Eq. (5), the constitutive relations for the unwound polymer matrix nanocomposite can be derived as

$$\{\sigma_{sc}\} = [C_{sc}]\{e_{sc}\} + \{\tau_{sc}\}$$  \hspace{1cm} (24)

in which the effective elastic coefficient matrix \([C_{sc}]\) and the effective coefficient of thermal expansion vector \([z_{sc}]\) of the unwound polymer matrix nanocomposite are given by

$$[C_{sc}] = \frac{1}{V} \sum_{\beta=1}^{M} \sum_{\gamma=1}^{N} V_{\beta \gamma} [C_{\beta \gamma}^p][A_{\beta \gamma}^p]$$

and

$$[z_{sc}] = -\frac{1}{V} \sum_{\beta=1}^{M} \sum_{\gamma=1}^{N} V_{\beta \gamma} [C_{\beta \gamma}^p] \left( [D_{\beta \gamma}^p] [z_{sc}] - \{e_{sp}\} \right)$$  \hspace{1cm} (25)

It may be noted that the effective elastic coefficient matrix \([C_{sc}]\) and the effective thermal expansion coefficient vector \([z_{sc}]\) directly provide the effective properties at a point in the portion of the polymer matrix nanocomposite surrounding the carbon fiber where the CNT is aligned with the \(3\)-axis. But for the point located in the polymer matrix nanocomposite where the CNT is oriented at an angle \(\theta\) with the \(3\)-axis in the \(2-3\) plane, \([C_{sc}]\) and \([z_{sc}]\) also provide the effective properties with respect to the local \((1', 2', 3')\) material coordinate system. Thus, the location-dependent effective elastic coefficient matrix \([C_{PMNC}]\) and the effective thermal expansion coefficient vector \([z_{PMNC}]\) at any point of the polymer matrix nanocomposite with respect to the \(1-2-3\) coordinate system can be obtained by the following transformations:

$$[C_{PMNC}] = [T]^{-1}[C_{sc}] [T]^{-1}$$

and

$$[z_{PMNC}] = [T]^{-1}[z_{sc}]$$  \hspace{1cm} (26)

where \(\{T\} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & m^2 & n^2 & mn & 0 \\ 0 & n^2 & m^2 & -mn & 0 \\ 0 & -2mn & 2mn & m^2 - n^2 & 0 \\ 0 & 0 & 0 & m & -n \\ 0 & 0 & 0 & n & m \end{bmatrix}\) with \(m = \cos \theta\) and \(n = \sin \theta\).

Therefore, the effective thermoelastic properties of the polymer matrix nanocomposite surrounding the carbon fiber with respect to the principle material coordinate axes of the proposed fuzzy fiber-reinforced composite varies over an annular cross-section of the polymer matrix nanocomposite phase of the RVE of the composite fuzzy fiber. However, without loss of generality, it may be considered that the volume average of these effective thermoelastic properties over the volume of the polymer matrix nanocomposite can be treated as the constant effective thermoelastic properties of the polymer matrix nanocomposite material surrounding the carbon fiber with respect to the \(1-2-3\) coordinate axes of the fuzzy fiber-reinforced composite and are given by

$$[C_{PMNC}] = \frac{1}{\pi} \int_{0}^{R} \{C_{PMNC}\} \mathbf{r} \, dr \, d\theta$$  \hspace{1cm} (27)

Thus, the effective constitutive relations for the polymer matrix nanocomposite material surrounding the carbon fiber with respect to the principle material coordinate axes of the fuzzy fiber-reinforced composite can be expressed as

$$\{\sigma_{PMNC}\} = [C_{PMNC}] \{e_{PMNC}\}$$  \hspace{1cm} (28)

Next, utilizing the thermoelastic properties of the polymer matrix nanocomposite as the matrix material properties and the carbon fiber, being aligned along the \(1\)-direction, as the reinforcement, the generalized method of cells model presented above for the unwound polymer matrix nanocomposite is suitably augmented to derive the effective thermoelastic properties of a lamina made of the composite fuzzy fiber. Finally, the generalized method of cells model for the composite fuzzy fiber is used in a straightforward manner to estimate the effective thermoelastic properties of the fuzzy fiber-reinforced composite lamina in which the monolithic polymer is the matrix material and the composite fuzzy fiber is the reinforcement along the \(1\)-direction.

### 2.2 Mori–Tanaka (MT) Method.

The generalized method of cells model derived in Sec. 2.1.1 is based on the assumptions delineated by Eqs. (6–17), which imply continuity of displacements and tractions between the fiber and the surrounding matrix material. For the purpose of verifying this model, the effective properties predicted by this model should be compared with those predicted by a different micromechanical model. Hence, another micromechanics model based on the Mori–Tanaka method [49] will be derived here. Utilizing the effective properties of the CNT and the polymer matrix properties, the Mori–Tanaka method [49] can be derived to estimate the effective elastic coefficient matrix of the unwound polymer matrix nanocomposite. The constitutive equations for the unwound polymer matrix nanocomposite material can be expressed as

$$\{\sigma_{sc}\} = [C_{sc}]\{e_{sc}\} + \{\tau_{sc}\}$$

and

$$\{e_{sc}\} = [M_{sc}]\{\sigma_{sc}\} + \{\tau_{sc}\}$$  \hspace{1cm} (29)

In Eq. (29), \([\tau_{sc}]\) and \([M_{sc}]\) denote, respectively, the thermal stress vector and the elastic compliance matrix of the unwound polymer matrix nanocomposite. The explicit formulation of the Mori–Tanaka model for the unwound polymer matrix nanocomposite material can be derived as

$$[C_{sc}] = [C_p] + v_n([C_{m}] - [C_p]) [A_1]$$  \hspace{1cm} (30)

in which the matrices of the strain concentration factors are given by

$$\{A_1\} = [A_1][v_p][I] + v_n[A_1]^{-1}$$

and

$$\{A_1\} = [I] + [S_1] [C_p]^{-1} [C_m] - [C_p]$$  \hspace{1cm} (31)

where \(v_n\) and \(v_p\) represent the volume fractions of the CNT fiber and the monolithic polymer material, respectively, present in the RVE of the polymer matrix nanocomposite, while \([S_1]\) represents...
the Eshelby tensor and the specific form of the Eshelby tensor for cylindrical inclusion given by Qui and Weng [50] is utilized. Using the effective elastic coefficient matrix \([\mathbf{C}^{\text{nc}}]\), the effective thermal expansion coefficient vector \([\alpha^{\text{nc}}]\) for the unwound polymer matrix nanocomposite material can be derived in the form [51] as follows:

\[
\{\alpha^{\text{nc}}\} = \{\alpha^p\} + \left( [\mathbf{C}^{\text{nc}}]^{-1} - [\mathbf{C}]^{-1}\right) \left( [\mathbf{C}]^{-1} - [\mathbf{C}^{\text{PMNC}}]^{-1}\right)^{-1} \\
\times \left( \{\alpha^p\} - \{\alpha^{\text{PMNC}}\} \right)
\]  

(32)

where \([\alpha^p]\) and \([\alpha^{\text{PMNC}}]\) are the thermal expansion coefficient vectors of the CNT fiber and the monolithic polymer material, respectively. Once \([\mathbf{C}^{\text{nc}}]\) and \([\alpha^{\text{nc}}]\) are computed, Eqs. (26) and (27) are used to estimate the average effective thermoelastic properties of the polymer matrix nanocomposite material surrounding the carbon fiber.

Since the composite fuzzy fiber is a composite in which the carbon fiber is the reinforcement and the matrix phase is the polymer matrix nanocomposite material, the Mori–Tanaka model can be employed to estimate its effective properties. Thus, according to the Mori–Tanaka model [49], the effective elastic coefficient matrix for the composite fuzzy fiber (CFF) is given by

\[
[\mathbf{C}^{\text{CFF}}] = [\mathbf{C}^{\text{PMNC}}] + \nu_p \left( [\mathbf{C}] - [\mathbf{C}^{\text{PMNC}}] \right) [\mathbf{A}_2]^{-1}
\]  

(33)

in which the matrices of the strain concentration factors are given by

\[
[\mathbf{A}_2] = \begin{bmatrix} \mathbf{A}_3 \end{bmatrix} [\mathbf{S}_2] \begin{bmatrix} \mathbf{A}_1 \end{bmatrix}^{-1} \text{ and } [\mathbf{A}_j] = \begin{bmatrix} \mathbf{A}_j \end{bmatrix} [\mathbf{S}_j] \begin{bmatrix} \mathbf{A}_1 \end{bmatrix}^{-1}
\]  

(34)

In Eqs. (33) and (34), \(\nu_p\) and \(\mathbf{S}_p\) are the volume fractions of the carbon fiber and the polymer matrix nanocomposite material, respectively, with respect to the volume of the RVE of the composite fuzzy fiber, and the Eshelby tensor \([\mathbf{S}_j]\) is computed based on the properties of the polymer matrix nanocomposite material and the shape of the carbon fiber. It is worthwhile to note that the polymer matrix nanocomposite is transversely isotropic and, consequently, the Eshelby tensor [52], corresponding to transversely isotropic material, is utilized for computing the matrix \([\mathbf{S}_2]\) while the inclusion is a circular cylindrical fiber. Using the effective elastic coefficient matrix \([\mathbf{C}^{\text{CFF}}]\), the effective thermal expansion coefficient vector for the composite fuzzy fiber can be derived as follows [51]:

\[
\{\alpha^{\text{CFF}}\} = \{\alpha^f\} + \left( [\mathbf{C}^{\text{CFF}}]^{-1} - [\mathbf{C}]^{-1}\right) \left( [\mathbf{C}]^{-1} - [\mathbf{C}^{\text{PMNC}}]^{-1}\right)^{-1} \\
\times \left( \{\alpha^f\} - \{\alpha^{\text{PMNC}}\} \right)
\]  

(35)

where \([\mathbf{C}^f]\) and \([\alpha^f]\) are the elastic coefficient matrix and the thermal expansion coefficient vector of the carbon fiber, respectively. Finally, considering the composite fuzzy fiber as the cylindrical inclusion embedded in the isotropic polymer matrix, the effective elastic properties \([\mathbf{C}^{\text{CFF}}]\) of the fuzzy fiber-reinforced composite can be derived by the Mori–Tanaka method [49] as follows:

\[
[\mathbf{C}] = [\mathbf{C}^p] + \nu_{\text{CFF}} \left( [\mathbf{C}^{\text{CFF}}] - [\mathbf{C}^p] \right) [\mathbf{A}_3]^{-1}
\]  

(36)

in which the matrices of the strain concentration factors are given by

\[
[\mathbf{A}_3] = \begin{bmatrix} \mathbf{A}_3 \end{bmatrix} [\mathbf{S}_3] \begin{bmatrix} \mathbf{A}_1 \end{bmatrix}^{-1} \text{ and } [\mathbf{A}_3] = \begin{bmatrix} \mathbf{A}_3 \end{bmatrix} [\mathbf{S}_3] \begin{bmatrix} \mathbf{A}_1 \end{bmatrix}^{-1} \text{ and } [\mathbf{A}_3] = \begin{bmatrix} \mathbf{A}_3 \end{bmatrix} [\mathbf{S}_3] \begin{bmatrix} \mathbf{A}_1 \end{bmatrix}^{-1}
\]  

(37)

where \(\nu_{\text{CFF}}\) and \(\nu_p\) are the volume fractions of the composite fuzzy fiber and the polymer material, respectively, with respect to the volume of the RVE of the fuzzy fiber-reinforced composite. Finally, the effective thermal expansion coefficient vector for the fuzzy fiber-reinforced composite can be derived as follows [51]:

\[
\{\alpha^{\text{CFF}}\} = \{\alpha^f\} + \left( [\mathbf{C}]^{-1} - [\mathbf{C}^{\text{CFF}}]^{-1}\right) \left( [\mathbf{C}]^{-1} - [\mathbf{C}^{\text{PMNC}}]^{-1}\right)^{-1} \\
\times \left( \{\alpha^f\} - \{\alpha^{\text{PMNC}}\} \right)
\]  

(38)

3 Results and Discussion

In this section, numerical values of the effective thermoelastic properties of the fuzzy fiber-reinforced composite are evaluated using the models derived in Secs. 2.1 and 2.2. Armchair single-walled CNTs, carbon fiber, and polymer matrix are considered for evaluating the numerical results. Their material properties have been taken from Refs. [53–55] and are listed in Table 1. Since the investigations by the earlier researchers [26–29] showed the strong temperature dependence of the coefficient of thermal expansion (CTE) of CNTs, the variation of CTEs of the armchair (10,10) CNT with the temperature deviation is considered here. But the elastic properties of CNTs, carbon fiber, and polymer are reported to be marginally dependent on temperature deviation [56]. Hence, the temperature dependence of the elastic properties of the constituent phases of the fuzzy fiber-reinforced composite is neglected. The relationships between the axial CTE \((\alpha^x_0)\) and the transverse CTE \((\alpha^t_0)\) of the armchair (10,10) CNT and the temperature deviation \((\Delta T)\) are given by [29]

\[
\alpha^x_0 = \alpha^x = 3.7601 \times 10^{-10} \Delta T^2 - 3.2189 \times 10^{-7} \Delta T - 3.2429 \times 10^{-8} \text{ K}^{-1}
\]  

(39)

\[
\alpha^t_0 = 6.4851 \times 10^{-11} \Delta T^2 - 5.8038 \times 10^{-8} \Delta T + 9.0295 \times 10^{-8} \text{ K}^{-1}
\]  

(40)

The value of the diameter of the carbon fiber is assumed as \(2a = 10 \mu m\). The volume fraction of CNTs \((\nu_{\text{CNT}})\) in the fuzzy fiber-reinforced composite depends on the CNT diameter, the carbon fiber diameter, and the surface-to-surface distance between two adjacent radially aligned CNTs at their roots. The surface-to-surface distance between the two adjacent CNTs at their roots is

<table>
<thead>
<tr>
<th>Material</th>
<th>(C_{11}) (GPa)</th>
<th>(C_{12}) (GPa)</th>
<th>(C_{13}) (GPa)</th>
<th>(C_{23}) (GPa)</th>
<th>(C_{33}) (GPa)</th>
<th>(C_{66}) (GPa)</th>
<th>(\alpha^x_0) ((10^{-6} \text{ K}^{-1}))</th>
<th>(\alpha^t_0) ((10^{-6} \text{ K}^{-1}))</th>
<th>(d_a = ) (\mu m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(10, 10) CNT, Ref. [7]</td>
<td>288</td>
<td>254</td>
<td>87.8</td>
<td>87.8</td>
<td>1088</td>
<td>17</td>
<td>*</td>
<td>*</td>
<td>1.36</td>
</tr>
<tr>
<td>Carbon fiber, Ref. [53, 54]</td>
<td>236.4</td>
<td>10.6</td>
<td>10.6</td>
<td>10.7</td>
<td>24.8</td>
<td>25</td>
<td>1.1</td>
<td>6.8</td>
<td>10,000</td>
</tr>
<tr>
<td>Polymer, Ref. [55]</td>
<td>4.09</td>
<td>1.55</td>
<td>1.55</td>
<td>1.55</td>
<td>4.09</td>
<td>1.27</td>
<td>66</td>
<td>66</td>
<td>—</td>
</tr>
</tbody>
</table>

\(\alpha_1\) and \(\alpha_2\) of the (10, 10) CNT are cited from Ref. [29] and presented in the text.
considered as 1.7 nm. Recall that the fuzzy fiber-reinforced composite lamina can be viewed as being comprised of the composite fuzzy fibers and the polymer matrix. For fibers with a circular cross-section, it is well known that hexagonal array of packing is the optimal packing of fibers, and the corresponding maximum fiber volume fraction is 0.9069. Hence, in the fuzzy fiber-reinforced composite, the hexagonal packing array of composite fuzzy fibers is considered, as shown in Fig. 7, for evaluating the reinforced composite, the maximum value of the CNT volume fraction (V\textsubscript{CNT})\textsubscript{max} present in the fuzzy fiber-reinforced composite with the carbon fiber volume fraction, while the values of \( v_f \) vary from 0.1 to \( \pi/2\sqrt{3} \). It may be observed from this figure that the value of V\textsubscript{CNT} is maximized at \( v_f = 0.24 \). In what follows, unless otherwise mentioned, the effective thermoelastic properties of the fuzzy fiber-reinforced composite are computed for a particular value of \( v_f \), while the maximum value of V\textsubscript{CNT} corresponding to this value of \( v_f \) is used for computing the numerical results.

In order to verify the validity of the micromechanics models derived in Secs. 2.1 and 2.2, the engineering constants of the unwound polymer matrix nanocomposite material determined by these models have been compared with those of the nanocomposite predicted by Liu and Chen [16]. Table 2 illustrates this comparison, and it may be observed that the three sets of results are in excellent agreement, validating the micromechanics models derived in this study. This agreement also ensures the validity of the assumptions adopted for the generalized method of cells approach.

First, the effective thermoelastic properties of the polymer matrix nanocomposite material surrounding the carbon fiber are computed by the models derived in Secs. 2.1 and 2.2. Figures 9 and 10 illustrate the variation of the effective thermal expansion coefficients \( \alpha_{\text{PMNC}} \) and \( \alpha_{\text{PMNC}}^{\text{PMNC}} \) of the polymer matrix nanocomposite, respectively, with the carbon fiber volume fraction. It may be observed that the thermal expansion coefficients predicted by the generalized method of cells approach excellently agree with those predicted by the Mori–Tanaka method for both the temperature variations. The value of \( \alpha_{\text{PMNC}}^{\text{PMNC}} \) does not vary with the increase in the temperature deviation, while that of \( \alpha_{\text{PMNC}}^{\text{PMNC}} \) slightly decreases with the same. Since the polymer matrix nanocomposite is transversely isotropic with 1-axis as the axis of symmetry, the values of the effective coefficients \( \alpha_{\text{PMNC}}^{\text{PMNC}} \) of the polymer matrix nanocomposite are found to be identical to those of \( \alpha_{\text{PMNC}}^{\text{PMNC}} \) but not presented here. The estimated effective thermoelastic properties of the polymer matrix nanocomposite are used to compute the effective thermoelastic properties of the composite fuzzy fiber and the fuzzy fiber-reinforced composite. However, for the sake of brevity, effective thermoelastic properties of the composite fuzzy fiber are not presented here.

Table 2 Comparison of the engineering constants of the unwound polymer matrix nanocomposite material

<table>
<thead>
<tr>
<th>( E_1/E_m^* )</th>
<th>( E_2/E_m )</th>
<th>( E_3/E_m )</th>
<th>( v_{23} )</th>
<th>( v_{12}, v_{13} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1.1948</td>
<td>1.1948</td>
<td>1.1837</td>
<td>0.04871</td>
</tr>
<tr>
<td>10</td>
<td>1.4384</td>
<td>1.4384</td>
<td>1.3316</td>
<td>0.4855</td>
</tr>
</tbody>
</table>

\( E_m = 1000 \text{ GPa}, v_m = 0.3, v_{23} = 0.3, \) and CNT volume fraction, \( v_n = 0.04871 \), where \( E_m \) and \( E_n \) are the Young’s modulus of the CNT and the polymer matrix, respectively; \( E_1 \) and \( E_2 \) are the axial Young’s modulus and the transverse Young’s modulus of the unwound polymer matrix nanocomposite, respectively; \( v_{12} \) and \( v_{23} \) are the Poisson’s ratio of the CNT and the polymer matrix, respectively; and \( v_{13} \) and \( v_n \) are the Poisson’s ratio of the unwound polymer matrix nanocomposite, respectively.

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Figure 11 illustrates the variation of the axial CTE ($a_1$) of the fuzzy fiber-reinforced composite with the carbon fiber volume fraction. It may be observed that the effective values of $a_1$ predicted by the generalized method of cells approach excellently agree with those predicted by the Mori–Tanaka method for both the temperature variations. It may be noted that the radial growing of CNTs on the carbon fibers does not appreciably influence the value of $a_1$ of the fuzzy fiber-reinforced composite. Figure 12 depicts the variation of the transverse CTE ($a_2$) of the fuzzy fiber-reinforced composite with the carbon fiber volume fraction. It may be noted that, for $V_{\text{CNT}} \neq 0$, the effective value of $a_2$ of the fuzzy fiber-reinforced composite is significantly lower than that of the composite without CNTs. This is attributed to the fact that the negative axial CTE ($a_n$) of the radially grown CNTs on the surface of the carbon fibers significantly suppresses the positive CTE ($a_m = 66 \times 10^{-6} \text{K}^{-1}$) of the polymer matrix, which eventually lowers the effective value of $a_2$ of the fuzzy fiber-reinforced composite. Although not shown here, the computed effective values of $a_1$ are found to match identically with that of $a_2$, corroborating that the fuzzy fiber-reinforced composite is transversely isotropic material. It is also important to note that, although the temperature variation locally affects the values of the CTEs of the polymer matrix nanocomposite material (Figs. 9 and 10), it does not significantly influence the effective values of the CTEs of the fuzzy fiber-reinforced composite. Since the two sets of the effective thermoelastic properties estimated by the generalized method of cells approach and the Mori–Tanaka method excellently agree with each other, as shown in Figs. 9–12, the subsequent results are estimated by the Mori–Tanaka approach for investigating effect of temperature deviation on the effective thermal expansion coefficients of the fuzzy fiber-reinforced composite.

Practically, the carbon fiber volume fraction in the composites can vary typically from 0.3 to 0.7. Hence, to analyze the effect of temperature deviation on the effective thermal expansion coefficients of the fuzzy fiber-reinforced composite, the four discrete values of the carbon fiber volume fraction are considered as 0.3, 0.4, 0.5, and 0.6. Figures 13 and 14 illustrate the variation of the effective axial CTE ($a_1$) and the transverse CTE ($a_2$) of the fuzzy fiber-reinforced composite with the temperature deviation, respectively, for different values of carbon fiber volume fraction. It is important to note from these figures that the temperature deviation has negligible effect on the thermal expansion coefficients of the fuzzy fiber-reinforced composite. It may also be observed that the increase in carbon fiber volume fraction in the fuzzy fiber-reinforced composite decreases the magnitude of the thermal expansion coefficients of the fuzzy fiber-reinforced composite.
axis. The axial thermal expansion coefficient \((\alpha_x)\) of the fuzzy fiber-reinforced composite is marginally affected by the radial growing of CNTs on the carbon fibers. Since the transverse thermal expansion coefficients of the fuzzy fiber-reinforced composite are significantly reduced, the proposed fuzzy fiber-reinforced composite will have better thermal stability in the transverse direction against the delamination failure. The present analysis may motivate the researchers for constructing this novel composite and serve the purpose of verifying the experimental estimates.

### Appendix

Equation (41), as shown in Sec. 3, can be derived as follows. Referring to Fig. 7, the RVE of the fuzzy fiber-reinforced composite can be considered as an equilateral triangle. Thus, the volume \((V_{\text{FFRC}})\) of the RVE of the fuzzy fiber-reinforced composite is given by

\[
V_{\text{FFRC}} = \frac{\sqrt{3}}{4} D^2 L
\]

where \(D = 2R\). The volume \((V^f)\) of the carbon fiber is

\[
V^f = \frac{\pi}{8} d^2 L
\]

where \(d = 2a\). Thus, the carbon fiber volume fraction \((v_f)\) in the fuzzy fiber-reinforced composite can be expressed as

\[
v_f = \frac{V^f}{V_{\text{FFRC}}} = \frac{\pi d^2}{2\sqrt{3} D^2}
\]

Using Eq. (A3), the carbon fiber volume fraction \((\bar{v}_f)\) in the composite fuzzy fiber can be derived as

\[
\bar{v}_f = \frac{\pi d^2 L}{8 D^2 L} \left(\frac{2\sqrt{3}}{\pi v_f} - v_f\right)
\]

The maximum number \((N_{\text{CNT}})_{\text{max}}\) of radially grown aligned CNTs on the surface of the carbon fiber is given by

\[
(N_{\text{CNT}})_{\text{max}} = \frac{\pi dL}{2(d_a + 1.7)^2}
\]

Therefore, the volume \((V_{\text{CNT}})\) of the CNTs is

\[
V_{\text{CNT}} = \frac{\pi d^2}{4} (R - a) (N_{\text{CNT}})_{\text{max}}
\]

Thus, the maximum volume fraction \((V_{\text{CNT}})_{\text{max}}\) of the CNT with respect to the volume of the fuzzy fiber-reinforced composite can be determined as

\[
(V_{\text{CNT}})_{\text{max}} = \frac{V_{\text{FFRC}}}{V_{\text{FFRC}}} = \frac{\pi d^2}{2(d_a + 1.7)^2} \left(\frac{\sqrt{\pi v_f}}{2\sqrt{3}} - v_f\right)
\]

Finally, the maximum volume fraction of the CNTs with respect to the volume of the polymer matrix nanocomposite \((v_n)_{\text{max}}\) and with respect to the volume of the composite fuzzy fiber \((\bar{v}_n)_{\text{max}}\) can be determined in terms of \((V_{\text{CNT}})_{\text{max}}\) as follows:

\[
(v_n)_{\text{max}} = \frac{V_{\text{CNT}}}{V_{\text{FFRC}}} = \frac{2\sqrt{3}}{\pi} \left(\frac{D^2}{D^2 - d^2}\right) (V_{\text{CNT}})_{\text{max}}
\]

\[
(\bar{v}_n)_{\text{max}} = \frac{V_{\text{CNT}}}{V_{\text{FF}} = \frac{2\sqrt{3}}{\pi} (V_{\text{CNT}})_{\text{max}}
\]

### Conclusion

Thermoelastic analysis of the fuzzy fiber-reinforced composite has been presented in this study. The fuzzy fiber-reinforced composite is composed of the single-walled armchair CNTs, the carbon fibers, and the polymer matrix. The carbon fiber reinforcements are horizontally aligned, and CNTs are radially grown on the surfaces of these carbon fibers. An analytical micromechanics model based on the generalized method of cells is derived to predict the effective thermoelastic properties of a laminate made of this composite. The Mori–Tanaka model is also derived for validating the assumptions adopted in the generalized method of cells model. The effective thermoelastic constants of the fuzzy fiber-reinforced composite predicted by the generalized method of cells model are found to be in excellent agreement with those predicted by the Mori–Tanaka model. Effects of temperature deviation on the effective CTEs of the fuzzy fiber-reinforced composite are found to be negligible; therefore, the fuzzy fiber-reinforced composite will have better thermal stability in the transverse direction against the delamination failure. The present analysis may motivate the researchers for constructing this novel composite and serve the purpose of verifying the experimental estimates.
References


