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What is This?
Smart damping of fuzzy fiber reinforced composite plates using 1–3 piezoelectric composites

Shailesh I Kundalwal\textsuperscript{1,2} and Manas Chandra Ray\textsuperscript{1}

Abstract
This article is concerned with the investigation of active constrained layer damping (ACLD) of smart laminated fuzzy fiber reinforced composite (FFRC) plates. The distinctive feature of the construction of this novel FFRC is that the uniformly spaced short carbon nanotubes (CNTs) are radially grown on the circumferential surfaces of carbon fibers. The effect of CNT waviness on the damping characteristics of the laminated FFRC plates is investigated when wavy CNTs are coplanar with either of the two mutually orthogonal planes. The constraining layer of the ACLD treatment is made of vertically/obliquely reinforced 1–3 piezoelectric composite material. A finite element model is developed for the laminated FFRC plates integrated with the patches of ACLD treatment. The effects of different boundary conditions of the FFRC plates and orientation angle of piezoelectric fibers on the damping characteristics of the laminated FFRC plates have also been investigated. Results reveal that if the plane of radially grown wavy CNTs on the circumferential surface of carbon fiber is coplanar with the plane of carbon fiber axis then the attenuation of amplitude of vibrations and the natural frequencies of the laminated FFRC plates are significantly improved over those of the FFRC containing straight CNTs or wavy CNTs being coplanar with the transverse plane of carbon fiber.

Keywords
1–3 piezoelectric composite, carbon nanotube waviness, finite element method, fuzzy fiber reinforced composite

1. Introduction
Monolithic piezoelectric materials have been extensively used as distributed sensors and actuators for developing smart structures with self-monitoring and controlling capabilities (Hwang and Park, 1993; Wang and Meguid, 2000; Ray and Baz, 2001; Trindade and Benjeddou, 2002; Garg and Anderson, 2003; Reddy and Ray, 2007; Ray and Pradhan, 2007; Ray et al., 2009; Sohn et al., 2009; Tian et al., 2009; Fakhari and Ohadi, 2010; Shah and Ray, 2012). Piezoelectric composites (PZCs) are usually composed of an epoxy matrix reinforced with the monolithic piezoelectric fibers providing a wide range of effective material properties which are not offered by the existing monolithic piezoelectric materials (Smith and Auld, 1991). One of the commercially available piezoelectric composites is a lamina of the vertically reinforced 1–3 PZC. In this 1–3 PZC lamina, the poling direction of piezoelectric fibers is along the thickness of the lamina while the top and bottom surfaces of the lamina are electrodeled. Micromechanical analyses (Smith and Auld, 1991; Ray and Pradhan, 2007) reveal that the magnitude of the effective out-of-plane piezoelectric coefficient (\(e_{33}\)) of this PZC is much larger than that of its effective in-plane piezoelectric coefficient (\(e_{31}\)). Hence, the in-plane actuation by this PZC lamina is negligible as compared to the out-of-plane actuation by the same. Ray and Pradhan (2007, 2008) investigated the performance of the vertically reinforced 1–3 PZC as the constraining layer of ACLD treatment for the active damping of...
smart composite structures. Baranyk et al. (2012) presented a semi-analytic approach for determining the influence of surface bonded piezoelectric actuators on the free vibration of a flexible beam. Shah and Ray (2012) investigated the performance of vertically/obliquely reinforced 1–3 PZC as the distributed actuators of a thin laminated composite truncated circular conical shell. Recently, an active control method has been proposed by Li and Narita (2013) to reduce the wind-induced vibration of the laminated plates by use of a velocity feedback control strategy. They reported that the amplitudes of the wind-induced vibration can be efficiently attenuated with much smaller control voltages by using a negative velocity feedback control strategy.

The research on the synthesis of molecular carbon structure by an arc-discharge method for evaporation of carbon led to the discovery of an extremely thin needle-like graphitic carbon nanotube (CNT; Iijima, 1991). Researchers probably thought that CNTs may be useful as nanoscale fibers for developing novel nano-composites, and this conjecture motivated them to accurately predict the elastic properties of CNTs. Many research investigations revealed that CNTs have axial Young’s modulus in the TeraPascal range (Treacy et al., 1996; Natsuki et al., 2004; Shen and Li, 2004). Three dimensional finite element (FE) models for armchair, zigzag and chiral single-walled CNTs have been developed by Tserpes and Papanikos (2005) to investigate the effects of CNT wall thickness, CNT diameter and chirality on the elastic moduli of single-walled CNTs. They varied the diameter of an armchair (8, 8) CNT between 0.066–0.34 nm and obtained the Young’s modulus in the range of 5.296–1.028 TPa. Batra and Gupta (2008) validated the mechanical response of a single-walled CNT with that of its equivalent continuum structure by defining the wall thickness as 1 Å and Young’s modulus as 3.3 TPa. Tsai et al. (2010) modeled the hollow cylindrical molecular structure of CNTs as an equivalent transversely isotropic solid cylinder and employed the molecular mechanics approach to determine the elastic properties of CNTs. They reported the values of axial Young’s modulus, transverse Young’s modulus and axial shear modulus of a zigzag (10, 0) CNT as 1382.5 GPa, 645 GPa and 1120 GPa, respectively. The quest for utilizing such remarkable mechanical properties of CNTs, their high aspect ratio and low density led to the emergence of a new area of research on the development of CNT-reinforced nanocomposites (Griebel and Hamaekers, 2004; Ashrafi and Hubert, 2006; Tsai et al., 2010). O’Donnell et al. (2004) investigated the potential application of CNT-reinforced composites for the airframes of commercial aircraft. In their study, it was reported that the structural mass of CNT-reinforced airframes is decreased approximately by 14.05% when compared with the aluminum airframes, which eventually decreases the fuel consumption by an average of 9.8% and increases the flight range by an average of 13.2%. Meguid et al. (2010) developed an atomistic-based continuum model to investigate the effective elastic properties of the CNT-reinforced composite. Their study revealed that the CNT length, volume fraction and orientation have significant effects on the effective elastic properties of CNT-reinforced composites.

Manufacturing of unidirectional CNT-reinforced composites in large scale encounters some challenging difficulties such as the agglomeration of CNTs, misalignment and difficulty in manufacturing very long CNTs (Thostenson et al., 2001; Wernik and Meguid, 2010; Meguid et al., 2013). Further research on improving the out-of-plane properties of composites and the better use of short CNTs led to the growth of short CNTs on the surfaces of advanced fibers. For example, Bower et al. (2000) have grown aligned CNTs on the substrate surface using high-frequency microwave plasma-enhanced chemical vapor deposition process. Qian et al. (2000) experimentally investigated the enhancement in the fiber/matrix interfacial shear strength being achieved by growing CNTs on the circumferential surfaces of fibers. Veedu et al. (2006) demonstrated that the remarkable improvements in the thermomechanical behavior of the laminated composite can be obtained by growing multi-walled CNTs about 60 μm long on the circumferential surfaces of fibers. The electrophoresis technique has been utilized for the selective deposition of CNTs on the surfaces of carbon fibers by Bekyarova et al. (2007). Garcia et al. (2008) fabricated a hybrid laminate in which the reinforcements are a woven cloth of alumina fibers with in situ grown CNTs on the circumferential surfaces of fibers. They demonstrated that the electro-mechanical properties of such a laminate are enhanced because CNTs were grown on the circumferential surfaces of alumina fibers. It seems that in comparison to the manufacturing and dispersion of long CNTs in the polymer matrix, direct growth of short CNTs on the circumferential surfaces of advanced fibers for achieving the uniform distribution of CNTs throughout a composite is practically more feasible and advantageous, and provides a plausible means to tailor the effective properties of the existing advanced fiber reinforced composites such that the structural benefits can be drawn from CNTs. Such a hybridized fiber augmented with radially grown CNTs on its circumferential surface is called a “fuzzy fiber” (Garcia et al., 2008), and the resulting composite may be called a fuzzy fiber reinforced composite (FFRC) (Kundalwal and Ray, 2011). Most recently, Kundalwal and Ray (2013) reported that
wavy CNTs can be properly grown on the circumferential surfaces of carbon fibers to improve the in-plane effective elastic properties of the continuous FFRC.

Carbon nanotube waviness is inherent to the fabrication process of CNT-reinforced polymer composites. Scanning electron microscopy (SEM) images shown in Figure 1 (Cao et al., 2005; Yamamoto et al., 2009; Zhang and Li, 2009) clearly demonstrate that CNTs remain highly curved when they are either embedded in the polymer matrix or grown on the circumferential surface of fiber.

Several studies reported that CNT waviness influences the effective elastic properties of micro- and nano-hybrid nanocomposites (Pantano and Cappello, 2008; Li and Chou, 2009; Tsai et al., 2011; Kundalwal and Ray, 2013). Based on a three-dimensional theory of elasticity, Jam et al. (2012) investigated the effects of CNT aspect ratio and waviness on the vibrational behavior of nanocomposite cylindrical panels, and their results indicate that the distribution pattern and volume fraction of CNTs have a significant effect on the natural frequencies of a nanocomposite cylindrical panel. A new concept for the optimization of dynamic behavior of the laminated nanocomposite beam is introduced by Rokni et al. (2012) in which fiber orientation factor in continuous fiber reinforced composites is replaced by different wt% of CNTs in each layer. This study revealed that the laminated nanocomposite beam with the optimum distribution of CNTs in the polymer matrix causes significant improvement in the effective Young’s modulus and damped natural frequencies over those of the beam with the uniformly reinforced CNTs. Based on a mesh-free method, Moradi-Dastjerdi (2013) investigated the influence of orientation and aggregation of CNTs on the axisymmetric natural frequencies of a functionally graded nanocomposite cylinder. They reported that the distribution pattern, aggregation or evenly random orientations of CNTs have a significant effect on the effective stiffness and frequency parameter of a functionally graded nanocomposite cylinder. Since the distribution pattern and CNT waviness influences the mechanical and vibrational behavior of the CNT-reinforced composite, the waviness of CNTs will also influence the damping characteristics of the laminated FFRC plates. Investigation of the effect of CNT waviness on the damping characteristics of the laminated FFRC plates is an important issue and has not yet been addressed. It is therefore the objective of this work to investigate the effect of CNT waviness on the vibrating properties.

Figure 1. (a) Scanning electron microscopy (SEM) image of a compressed carbon nanotube (CNT)-reinforced film. Reproduced with kind permission from AAAS (Cao et al., 2005); (b) SEM image of the alumina fiber on which radially aligned CNTs are grown. Reproduced with kind permission from Elsevier (Yamamoto et al., 2009); (c) and (d) SEM images of a multi-walled CNT array with wavy structure at different magnifications. Reproduced with kind permission from Elsevier (Zhang and Li, 2009).
thin laminated FFRC plates integrated with the ACLD patches. The waviness of CNTs is accounted for by considering the plane of wavy CNTs as coplanar with either the longitudinal plane or the transverse plane of the carbon fiber. Using the FSDT, a three-dimensional FE model is developed to study the damping characteristics of the laminated FFRC plates. The effects of piezoelectric fibers' orientation in either of the two mutually orthogonal vertical planes of the PZC layer and different boundary conditions of the FFRC plates on the damping characteristics of the laminated FFRC plates have also been studied.

2. Nanocomposite containing aligned CNTs with different shapes

Carbon nanotubes with different tubule morphologies have their own special properties and potential applications. A single CNT naturally curves (in bending status) during growth if no external forces exist. In principle, the CNT bending (kinks) can originate from a pentagon-pentagon topological defect pair or a local mechanical deformation in a uniform CNT. During growth, the bending stress can come from the CNT's own weight, interaction with neighbor CNTs, or limited growing space (Zhang and Li, 2009). For the first time, Cao et al. (2005) exhibited super-compressible foamlke behavior of freestanding films of vertically aligned CNTs. They reported that compared with conventional low-density flexible foams, the CNT films show much higher compressive strength, recovery rate and sage factor, and the open-cell nature of the CNT arrays gives excellent breathability. SEM image (see Figure 1(a)) of the film of the thickness 720 μm shows ordered wavelike folds along CNTs which are formed across the film section. Zhang and Li (2009) reviewed different shapes of CNTs formed during growth, their morphologies and possible applications. For example, Figure 1(c) and (d) show a CNT array, in which more than 80% of CNTs are not straight; they periodically bend within fixed intervals throughout their entire length. As a result of this regular bending, a wavy structure is formed. Recently, Wardle et al. (2009, 2011, 2013) fabricated aligned CNT composite by implementing a controlled and aligned morphology via a chemical vapor deposition technique. In brief, to form the aligned CNT composite, degassed aerospace-grade epoxy was heated to 90 °C. The CNT forest was then placed on top of the epoxy, which then slowly infused the forest due to capillary forces. The cured aligned CNT composite block was then placed into a silicone dogbone mold in the desired tensile testing orientation such that alignment of CNTs was coplanar with either the parallel or perpendicular axis of dogbone (Handlin et al., 2013). Subsequently, Handlin et al. (2013) developed the elastic constitutive relations for the aligned CNT composite with maximum 18% CNT volume fraction to study the effect of CNT waviness and found that CNT waviness has a very large impact on the aligned CNT composite. The review of literature on the aligned CNT arrays reveals that CNT waviness is intrinsic to manufacturing processes and may occur in the two mutually orthogonal planes if aligned CNT films are fabricated using well-controlled growth morphology. Hence, two possible planar orientations of wavy CNTs are considered either in the transverse (2–3) plane or the longitudinal (1–3) plane of carbon fiber, as shown in Figure 2(a) and (b), respectively, to investigate the effect of CNT

Figure 2. (a) Fuzzy fiber coated with wavy carbon nanotubes (CNTs) being coplanar with the transverse (2–3) plane of the carbon fiber; (b) Fuzzy fiber coated with wavy CNTs being coplanar with the longitudinal (1–3) plane of the carbon fiber.
waviness on the damping characteristics of the laminated FFRC plates.

2.1. Architecture of the FFRC containing wavy CNTs

The schematic illustrated in Figure 3 represents a lamina of the FFRC containing wavy CNTs being studied here. The distinctive feature of the construction of such novel composite is that wavy CNTs are radially grown on the circumferential surfaces of long carbon fibers, while they are uniformly spaced on the circumferential surface of carbon fiber. The radially grown wavy CNTs eventually reinforce the polymer matrix surrounding the carbon fiber along the transverse direction to the length of the carbon fiber. Thus, the combination of the fuzzy fiber with wavy CNTs and the polymer matrix can be viewed as a circular cylindrical composite fuzzy fiber (CFF) in which a carbon fiber is embedded in wavy CNT-reinforced polymer matrix nanocomposite (PMNC). It should be noted that the variations of the constructional feature of the CFF can be such that the waviness of CNTs is coplanar with either the 2–3 plane or the 1–3 plane as shown in Figure 4(a) and (b), respectively.

3. Theoretical formulation

In this section, FE model is developed for analyzing the ACLD of the laminated FFRC plates comprising N number of laminae. Such laminated FFRC plate being integrated with the ACLD patches on its top surface is illustrated in Figure 5. The constructional feature of the ACLD patches can be such that piezoelectric fibers are coplanar with either the xz plane or the yz plane of the PZC layer as shown in Figure 6(a) and (b), respectively. The piezoelectric fiber orientation in the PZC layer with respect to the z axis is denoted by $\psi$. The thickness of the PZC layer is $h_p$ and that of the viscoelastic layer of the ACLD treatment is $h_v$. The middle plane of the substrate FFRC plate is considered as the reference plane. The origin of the laminate coordinate system $(xyz)$ is located on the reference plane in such a way that the lines $x = 0$, $a$ and $y = 0$, $b$ represent the boundaries of the substrate plates. The thickness coordinates $(z)$ of the top and bottom surfaces of any $(k$th) layer of the overall plate are represented by $h_{k+1}$ and $h_k(k = 1, 2, 3, \ldots, N + 2)$, respectively.

3.1. Displacement fields

Figure 7 illustrates the kinematics of axial deformations of the overall plate based on the first-order shear deformation theory.

As shown in Figure 7, $u_0$ and $v_0$ are the generalized translational displacements of a reference point $(x, y)$ on the mid-plane $(z = 0)$ of the substrate composite plate along the $x$- and $y$-directions, respectively; $\theta_x$, $\phi$, and $\gamma_x$ are the generalized rotations of the normals to the middle planes of the substrate composite plates, viscoelastic layer and PZC layer, respectively in the xz plane, while $\theta_y$, $\phi_y$ and $\gamma_y$ represent their generalized rotations in the yz plane. Following the first-order shear deformation theory, the axial displacements...
\[ u(x, y, z, t) = u_0(x, y, t) + (z - (z - h/2))\phi_5(x, y, t) + \left( (z - h/2) - (z - h_{N+2}) \right)\psi_3(x, y, t) + \left( (z - h_{N+2}) \right)\phi_5(x, y, t) \]  
\[ v(x, y, z, t) = v_0(x, y, t) + \left( z - (z - h/2) \right)\delta_5(x, y, t) + \left( (z - h/2) - (z - h_{N+2}) \right)\phi_5(x, y, t) + \left( (z - h_{N+2}) \right)\psi_3(x, y, t) \]  
in which a function within the bracket \( () \) represents the appropriate singularity functions. Since the transverse

**Figure 4.** Transverse and longitudinal cross sections of the composite fuzzy fiber (CFF) in which wavy carbon nanotubes (CNTs) are coplanar with either the 2–3 or the 1–3 plane. (a) Cross sections of the CFF with wavy CNTs being coplanar with the 2–3 plane; (b) Cross sections of the CFF with wavy CNTs being coplanar with the 1–3 plane.

**Figure 5.** Schematic representation of laminated fuzzy fiber reinforced composite (FFRC) plates integrated with the active constrained layer damping patches composed of the vertically reinforced 1–3 PZC constraining layer.
normal strain is usually considered as negligible for thin structures, the radial displacement \( w \) is assumed to be linearly varying across the thickness of the substrate composite plate, viscoelastic layer and PZC layer. Consequently, the transverse displacement at any point in the overall plate can be assumed as

\[
w(x, y, z, t) = w_0(x, y, t) + \frac{z}{C_0} \phi_z(x, y, t) + \frac{z}{C_10/C_11/C_0/C_1} \psi_0(x, y, t) + \frac{z}{C_0} h_N + \frac{z}{C_10/C_11/C_0/C_1} \phi_z(x, y, t)^3 \]

in which \( w_0 \) refers to the transverse displacement at any point on the reference plane; \( \phi_z \) and \( \psi_0 \) are the generalized displacements representing the gradients of the transverse displacement in the substrate composite plates, viscoelastic layer and PZC layer, respectively with respect to the thickness coordinate \( z \).

For convenience, the generalized displacement variables are separated into generalized translational \( \{d_t\} \) and rotational \( \{d_r\} \) displacements as follows:

\[
\{d_t\} = [u_0 \ v_0 \ w_0]^T \quad \text{and} \quad \{d_r\} = [\theta_x \ \theta_y \ \theta_z \ \phi_x \ \phi_y \ \phi_z \ \gamma_x \ \gamma_y \ \gamma_z]^T \quad (4)
\]

### 3.2. Strain-displacement relations

In order to implement the selective integration rule for avoiding the so-called shear locking problem in the thin laminated FFRC shell, the state of strains at any point in the overall FFRC shell is grouped into two strain vectors separating the transverse shear strains as follows:

\[
\{\varepsilon_h\} = [\varepsilon_x \ \varepsilon_y \ \varepsilon_{xy}]^T \quad \text{and} \quad \{\varepsilon_s\} = [\varepsilon_{xz} \ \varepsilon_{yz}]^T \quad (5)
\]

---

**Figure 6.** Schematic diagram of the lamina of the vertically reinforced 1–3 PZCs in which piezoelectric fibers are coplanar with either the vertical xz or yz plane. (a) Piezoelectric fibers are coplanar with the vertical xz plane; (b) Piezoelectric fibers are coplanar with the vertical yz plane.

**Figure 7.** Kinematics of deformation.
where \( \varepsilon_x \), \( \varepsilon_y \) and \( \varepsilon_z \) are the normal strains along the \( x \), \( y \) and \( z \) directions, respectively; \( \varepsilon_{xy} \) is the in-plane shear strain; \( \varepsilon_{xz} \) and \( \varepsilon_{yz} \) are the transverse shear strains. Using the linear strain-displacement relations, the displacement field equations (1)–(3) and equation (5), the vectors \( \{e_b\}_c \), \( \{e_s\}_c \) and \( \{e_b\}_p \) defining the state of in-plane and transverse normal strains at any point in the substrate composite plate, viscoelastic layer and PZC layer, respectively, can be expressed as

\[
\{e_b\}_c = \{e_b\}_c + [Z]\{e_b\}_c, \quad \{e_s\}_c = \{e_s\}_c + [Z]\{e_s\}_c, \quad \{e_b\}_p = \{e_b\}_p + [Z]\{e_b\}_p
\]

(6)

whereas the vectors \( \{e_b\}_c \), \( \{e_s\}_c \) and \( \{e_b\}_p \) defining the state of transverse shear strains at any point in the substrate composite plate, viscoelastic layer and PZC layer, respectively, can be expressed as

\[
\{e_s\}_c = \{e_s\}_c + [Z]\{e_s\}_c, \quad \{e_s\}_s = \{e_s\}_s + [Z]\{e_s\}_s
\]

(7)

The various matrices appearing in equations (6) and (7) are defined in Appendix A, while the generalized strain vectors are given by

\[
\{e_b\}_c = \left[ \begin{array}{c} \varepsilon_{xx} \\
\varepsilon_{yy} \\
\varepsilon_{zz} + \alpha_{xx} + \alpha_{yy} - \alpha_{zz} \end{array} \right], \quad \{e_s\}_c = \left[ \begin{array}{c} \varepsilon_{xy} \\
\varepsilon_{yx} \\
\theta_x + \phi_x \end{array} \right], \quad \{e_b\}_p = \left[ \begin{array}{c} \varepsilon_{xx} \\
\varepsilon_{yy} \\
\varepsilon_{zz} + \alpha_{xx} + \alpha_{yy} - \alpha_{zz} \end{array} \right]
\]

(8)

and

\[
\{e_s\}_s = \left[ \begin{array}{c} \theta_x + \phi_x \\
\theta_y + \phi_y \\
\gamma_x + \gamma_y \end{array} \right]
\]

3.3. Constitutive relations

Corresponding to the description of the states of strains, the state of stresses at any point in the overall plate continuum are described by the state of in-plane and out-of-plane stresses \( \sigma_b \), and the state of transverse shear stresses \( \sigma_s \) as follows:

\[
\{\sigma_b\} = \begin{bmatrix} \sigma_x & \sigma_y & \sigma_{xy} \\ \sigma_{yx} & \sigma_y & \sigma_{yx} \end{bmatrix}^T \quad \text{and} \quad \{\sigma_s\} = \begin{bmatrix} \sigma_{xz} & \sigma_{yz} \end{bmatrix}^T
\]

(9)

where \( \sigma_x \), \( \sigma_y \) and \( \sigma_z \) are the normal stresses along the \( x \), \( y \) and \( z \) directions, respectively; \( \sigma_{xy} \) is the in-plane shear stress; \( \sigma_{xz} \) and \( \sigma_{yz} \) are the transverse shear stresses.

The constitutive relations for the material of any orthotropic layer of the substrate composite plate are given by

\[
\{\sigma_b^{k}\} = [C_k^{b}]\{e_b^{k}\} \quad \text{and} \quad \{\sigma_s^{k}\} = [C_k^{s}]\{e_s^{k}\}; \quad (k = 1, 2, 3, \ldots, N)
\]

(10)

where

\[
[C_k^{b}] = \begin{bmatrix}
C_{11}^{b} & C_{12}^{b} & C_{13}^{b} \\
C_{12}^{b} & C_{22}^{b} & C_{23}^{b} \\
C_{13}^{b} & C_{23}^{b} & C_{33}^{b}
\end{bmatrix}, \quad [C_k^{s}] = \begin{bmatrix}
C_{55}^{s} & C_{54}^{s} \\
C_{54}^{s} & C_{44}^{s}
\end{bmatrix}
\]

in which \( C_k^{b} \) (i, j = 1, 2, 3, \ldots, 6) are the transformed elastic coefficients with respect to the reference coordinate system.

The constraining PZC layer will be subjected to the applied electric field acting along its thickness direction only. Thus, the constitutive relations for the material of the PZC layer can be expressed as (Ray and Pradhan, 2007)

\[
\{\sigma_b^{k}\} = [C_k^{b}]\{e_b^{k}\} + [C_k^{b}]\{e_b^{k}\} - [e_b]E_z, \quad \{\sigma_s^{k}\} = [C_k^{s}]\{e_s^{k}\} + [C_k^{s}]\{e_s^{k}\} - [e_s]E_z \quad \text{and}
\]

\[
D_z = [E_b]^T\{e_b^{k}\} + [e_b]^T\{e_b^{k}\} + \tilde{e}_{33}E_z, \quad k = N + 2
\]

(11)

in which \( E_z \) and \( D_z \) are the applied electric and its displacement along the z-direction, respectively, and \( \tilde{e}_{33} \) is the dielectric constant. It may be noted from the above constitutive relations that the transverse shear strains are coupled with the in-plane strains due to the orientation of piezoelectric fibers in the vertical xz or yz plane of the PZC layer and the corresponding coupling elastic coefficients matrices \( [C_{N+2}^{b}] \) are given by

\[
[C_{N+2}^{b}] = \begin{bmatrix}
C_{15}^{N+2} & C_{25}^{N+2} & 0 & \tilde{C}_{35}^{N+2} \\
0 & 0 & \tilde{C}_{46}^{N+2} & 0 \\
0 & 0 & \tilde{C}_{56}^{N+2} & 0 \\
0 & 0 & \tilde{C}_{46}^{N+2} & 0
\end{bmatrix}^T
\]

(12)

or

\[
[C_{N+2}^{s}] = \begin{bmatrix}
0 & 0 & \tilde{C}_{56}^{N+2} & 0 \\
0 & 0 & \tilde{C}_{35}^{N+2} & 0 \\
0 & 0 & \tilde{C}_{46}^{N+2} & 0 \\
0 & 0 & \tilde{C}_{56}^{N+2} & 0
\end{bmatrix}^T
\]

(13)

corresponding to piezoelectric fibers being coplanar with the vertical xz or yz plane, respectively, in the PZC layer. Also, the piezoelectric constant matrices \( [e_b] \) and \( [e_s] \) appearing in equation (11) referred to the reference coordinate system (xyz); such that,

\[
[e_b] = \begin{bmatrix}
\tilde{e}_{31} & \tilde{e}_{32} & \tilde{e}_{36} & \tilde{e}_{33}
\end{bmatrix}^T \quad \text{and} \quad [e_s] = \begin{bmatrix}
\tilde{e}_{31} & \tilde{e}_{32} & \tilde{e}_{33}
\end{bmatrix}^T
\]
The present analysis is concerned with the frequency response of the plates integrated with the patches of ACLD treatment. Hence, the viscoelastic material is modeled by using the complex modulus approach. The material of the viscoelastic layer is assumed to be linearly viscoelastic and isotropic, while the shear modulus (G) and the Young’s modulus (E) of the viscoelastic material are given by (Shen, 1996):

\[ G = G'(1 + i\eta) \quad \text{and} \quad E = 2G(1 + \nu) \quad (14) \]

where \( G' \) is the storage modulus, \( \nu \) is the Poisson’s ratio and \( \eta \) is the loss factor at a particular operating temperature and frequency. Using equation (14), the elastic coefficients of the viscoelastic material can be determined and the resulting elastic coefficients matrix \( C_{N+1}^{(N+1)} \) turns out to be a complex matrix (Shen, 1996; Ray and Baz, 1997; Jeung and Shen, 2001).

### 3.4. Finite element modeling

The principle of virtual work is employed to derive the governing equations of the overall FFRC plate/ACLD system (Jeung and Shen, 2001); such that,

\[
\sum_{k=1}^{N+2} \int _{\Omega} \left( [\epsilon_k^f] ^T [\sigma_k^f] + \delta [\epsilon_k^f]^T \delta [\sigma_k^f] - \delta E_z \delta [\varepsilon_{z13}] \right) \text{d}\Omega - \int _{\Omega} \delta [d_i^f]^T \{ P \} \text{d}A = 0 \quad (15)
\]

in which \( \rho^k \) is the mass density of the \( k \)th layer, \( \Omega \) is the externally applied surface traction acting over a surface area (A) and \( \varepsilon_{z13} \) represents the volume of the concerned layer.

Eight-noded isoparametric quadrilateral elements have been used to discretize the overall plate. According to equation (4), the generalized displacement vectors associated with the \( i \)th (\( i = 1, 2, 3, \ldots, 8 \)) node of the element can be written as

\[
\begin{align*}
d_i &= \begin{bmatrix} u_{i1} & v_{i1} & w_{i1} \end{bmatrix}^T \\
d_r &= \begin{bmatrix} \theta_{i1} & \theta_{i2} & \phi_{i1} & \phi_{i2} & \gamma_{i1} & \gamma_{i2} \end{bmatrix}^T
\end{align*}
\]

Thus, the generalized displacement vectors \( \{d_i\} \) and \( \{d_r\} \) at any point within the element can be expressed in terms of the nodal generalized displacement vectors \( \{d_i^e\} \) and \( \{d_r^e\} \) as follows:

\[
\begin{align*}
\{d_i\} &= \{N_i\} \{d_i^e\} \quad \text{and} \\
\{d_r\} &= \{N_r\} \{d_r^e\} \quad (17)
\end{align*}
\]

in which

\[
\begin{align*}
\{N_i\} &= \begin{bmatrix} N_{i1} & N_{i2} & \cdots & N_{i8} \end{bmatrix}^T, \\
\{N_r\} &= \begin{bmatrix} N_{r1} & N_{r2} & \cdots & N_{r8} \end{bmatrix}^T, \\
N_{ri} &= n_i I_r,
\end{align*}
\]

\[
\begin{align*}
N_{ri} &= n_i I_r \{d_i^e\} = \begin{bmatrix} \{d_i^e\}^T & \{d_i^e\}^T & \cdots & \{d_i^e\}^T \end{bmatrix}^T \\
\{d_i^e\} &= \begin{bmatrix} \{d_i^e\}^T & \{d_i^e\}^T & \cdots & \{d_i^e\}^T \end{bmatrix}^T
\end{align*}
\]

wherein \( I_r \) and \( I_t \) are \((3 \times 3)\) and \((9 \times 9)\) identity matrices, respectively, and \( n_i \) is the shape function of the natural coordinates associated with the \( i \)th node. Using equations (6)–(8) and (17), the strain vectors at any point within the element can be expressed in terms of the nodal generalized displacement vectors as follows:

\[
\begin{align*}
\{\varepsilon_k^f\} &= \{B_{ts}\} \{d_i^e\} + \{Z_1\} \{B_{ts}\} \{d_i^e\}, \\
\{\varepsilon_k^f\} &= \{B_{ts}\} \{d_i^e\} + \{Z_2\} \{B_{ts}\} \{d_i^e\}, \\
\{\varepsilon_k^f\} &= \{B_{ts}\} \{d_i^e\} + \{Z_3\} \{B_{ts}\} \{d_i^e\}, \\
\{\varepsilon_k^f\} &= \{B_{ts}\} \{d_i^e\} + \{Z_4\} \{B_{ts}\} \{d_i^e\} \quad (19)
\end{align*}
\]

while the nodal strain-displacement matrices \( \{B_{ts}\}, \{B_{rb}\}, \{B_{rs}\} \) and \( \{B_{ts}\} \) are given by

\[
\begin{align*}
\{B_{ts}\} &= \begin{bmatrix} B_{ts1} & B_{ts2} & \cdots & B_{ts8} \end{bmatrix}^T, \\
\{B_{rb}\} &= \begin{bmatrix} B_{rb1} & B_{rb2} & \cdots & B_{rb8} \end{bmatrix}^T, \\
\{B_{rs}\} &= \begin{bmatrix} B_{rs1} & B_{rs2} & \cdots & B_{rs8} \end{bmatrix}^T, \\
\{B_{ts}\} &= \begin{bmatrix} B_{ts1} & B_{ts2} & \cdots & B_{ts8} \end{bmatrix}^T
\end{align*}
\]

The elements of matrices \( \{B_{ts}\}, \{B_{rb}\}, \{B_{rs}\} \) and \( \{B_{ts}\} \) are presented in Appendix A. On substitution of equations (11) and (19) into equation (15), and recognizing that \( E_z = V/l_h \) with \( V \) being the applied voltage across the thickness of the PZC layer, the following open-loop equations of motion of an element integrated with the ACLD treatment can be derived:

\[
\begin{align*}
[M^f]\{\ddot{d}_i^e\} + [K_t^e]\{d_i^e\} + [K_p^e]\{d_i^e\} &= [K_{ip}^e]V + [F_p] \quad (21) \\
[K_t^e]\{d_i^e\} + [K_p^e]\{d_i^e\} &= [K_{ip}^e]V \quad (22)
\end{align*}
\]

The elemental mass matrix \( M^f \); the elemental stiffness matrices \( K_t^e, K_p^e, K_p^f \), the elemental electro-elastic coupling vectors \( F_p, F_p^f \); the elemental load
vector \( \{F^e\} \) and the mass parameter (\( \bar{m} \)) appearing in equations (21) and (22) are given by

\[
[M^e] = \int_a^b \int_0^b \bar{m}[N_i]^T[N_i]dx
dy,
\]

\[
[K^e_{ib}] = [K^e_{ib}] + [K^e_{ib}] + [K^e_{ib}]_{pb} + [K^e_{ib}]_{ps},
\]

\[
[K^e_{ts}] = [K^e_{ts}] + [K^e_{ts}] + [K^e_{ts}]_{pb} + [K^e_{ts}]_{ps},
\]

\[
[F^e_{ip}] = [F^e_{ip}] + [F^e_{ip}] + [F^e_{ip}]_{ps} \quad \text{and} \quad \{F^e\} = \int_a^b \int_0^b [N_i]^Tf\ dx\ dy \quad \text{and} \quad \bar{m} = \sum_{k=1}^{N_t-1} \rho^k(\rho_{k+1} - \rho_k) \quad (23)
\]

The various elemental stiffness matrices and the electroelastic coupling vectors appearing in equation (23) are as follows:

\[
[K^e_{ib}] = \int_A [B^T_{ib}]^T\{D_{ib}\} + [D_{ib}]_{ps} + [D_{ib}]_{ps}\{B_{ib}\}\ dx\ dy,
\]

\[
[K^e_{ts}] = \int_A [B^T_{ts}]^T\{D_{ts}\} + [D_{ts}]_{ps} + [D_{ts}]_{ps}\{B_{ts}\}\ dx\ dy,
\]

\[
[K^e_{ib}]_{ps} = \int_A [B^T_{ib}]^T\{D_{ib}\}_{ps}\{B_{ib}\}\ dx\ dy,
\]

\[
[K^e_{ts}]_{ps} = \int_A [B^T_{ts}]^T\{D_{ts}\}_{ps}\{B_{ts}\}\ dx\ dy.
\]

Also, the various rigidity matrices and vectors appearing in the above elemental matrices are given by

\[
[D_{ib}] = \sum_{k=1}^{N_t} \int_{h_k}^{h_{k+1}} [\hat{C}_b^k]dz, \quad [D_{trb}] = \sum_{k=1}^{N_t} \int_{h_k}^{h_{k+1}} [\hat{C}_s^k][Z_1]dz, \quad [D_{rb}] = \sum_{k=1}^{N_t} \int_{h_k}^{h_{k+1}} [Z_1]^T[\hat{C}_b^k]dz,
\]

\[
[D_{ts}] = \sum_{k=1}^{N_t} \int_{h_k}^{h_{k+1}} [\hat{C}_b^k]dz, \quad [D_{mts}] = \sum_{k=1}^{N_t} \int_{h_k}^{h_{k+1}} [\hat{C}_s^k][Z_4]dz, \quad [D_{ms}] = \sum_{k=1}^{N_t} \int_{h_k}^{h_{k+1}} [Z_4]^T[\hat{C}_b^k][Z_4]dz,
\]

\[
[D_{h,s}] = h_s[\hat{C}_b^{N+1}], \quad [D_{h,b}] = \int_{h_{N+2}}^{h_{N+2}} [\hat{C}_b^{N+1}][Z_2]dz,
\]

\[
[D_{rb}] = \int_{h_{N+2}}^{h_{N+2}} [Z_2]^T[\hat{C}_b^{N+1}][Z_2]dz, \quad [D_{h,s}] = h_s[\hat{C}_s^{N+1}],
\]

\[
[D_{ts}] = \int_{h_{N+2}}^{h_{N+2}} [\hat{C}_s^{N+1}][Z_5]dz, \quad [D_{h,s}] = \int_{h_{N+2}}^{h_{N+2}} [Z_5]^T[\hat{C}_s^{N+1}][Z_5]dz, \quad [D_{h,b}] = h_b[\hat{C}_b^{N+2}],
\]

\[
[D_{ib}] = \int_{h_{N+2}}^{h_{N+2}} [B^T_{ib}]^T\{D_{ib}\}_{ps} + [D_{ib}]_{ps} + [D_{ib}]_{ps}\{B_{ib}\}\ dx\ dy,
\]

\[
[K^e_{ib}]_{ps} = \int_A [B^T_{ib}]^T\{D_{ib}\}_{ps} + [D_{ib}]_{ps} + [D_{ib}]_{ps}\{B_{ib}\}\ dx\ dy.
\]

\[
[K^e_{ts}]_{ps} = \int_A [B^T_{ts}]^T\{D_{ts}\}_{ps} + [D_{ts}]_{ps} + [D_{ts}]_{ps}\{B_{ts}\}\ dx\ dy.
\]
control voltage supplied to the control law has been used. According to this law, the patches of ACLD treatment, a simple velocity feedback as assembled to obtain the open-loop global equations of motion of the overall plate integrated with the ACLD patches as follows:

\[
\begin{align*}
[M][\ddot{X}] + [K_u][X] + [K_t][X_f] &= \sum_{j=1}^{q} \left\{F_{ip}^{j}\right\} V^j + \{F\} \\
\end{align*}
\]

and

\[
[K_t][X] + [K_{trt}][X_r] + \sum_{j=1}^{q} \left\{F_{tr}^{j}\right\} V^j
\]

where \([M]\) is the global mass matrix; \([K_u]\), \([K_t]\), \([K_{tr}^c]\) and \([K_{tr}^s]\) are the global stiffness matrices; \(X\) and \(X_f\) are the global nodal translational and rotational degrees of freedom; \(F_{ip}^{j}\) and \(F_{tr}^{j}\) are the global electro-elastic coupling vectors corresponding to the \(j\)th patch; \(V^j\) is the voltage applied to this patch; \(q\) is the number of patches and \(\{F\}\) is the global nodal force vector.

4. Closed-loop model

In order to supply the control voltage for activating the patches of ACLD treatment, a simple velocity feedback control law has been used. According to this law, the control voltage supplied to the \(j\)th patch can be expressed in terms of the derivatives of the global nodal degrees of freedom; such that,

\[
V^j = -k_d^j \ddot{w} = -k_d^j \left\{U_r^j\right\}[\ddot{X}] - k_d^j (h/2) \left\{U_r^j\right\}[\dddot{X}]_r
\]

Finally, the elemental equations of motion are obtained by the FE model derived in the preceding section have been presented. Laminated symmetric/antisymmetric cross-ply and antisymmetric angle-ply thin square FFRC plates integrated with the two patches of ACLD treatment are considered for presenting the numerical results. It should be noted that the locations of the ACLD patches correspond to an optimal placement of the ACLD treatment selected such that the energy dissipation of the first two modes becomes maximum (Ro and Baz, 2002). To analyze the damping characteristics of

5. Results and discussion

In this section, the numerical results obtained by the FE model derived in the preceding section have been presented. Laminated symmetric/antisymmetric cross-ply and antisymmetric angle-ply thin square FFRC plates integrated with the two patches of ACLD treatment are considered for presenting the numerical results. It should be noted that the locations of the ACLD patches correspond to an optimal placement of the ACLD treatment selected such that the energy dissipation of the first two modes becomes maximum (Ro and Baz, 2002). To analyze the damping characteristics of
the laminated FFRC plates, the carbon fiber diameter and its volume fraction in the FFRC are taken as 10 μm and 0.6, respectively. The diameter of the CFF (2R) and the length of straight CNT (L₀) become 12.2943 μm and 1.1471 μm, respectively, when the value of \( \nu_T \) is 0.6 (Kundalwal and Ray, 2013). The maximum amplitude for zigzag (10, 0) CNT is considered as \( \text{amplitude (A)} \) and waviness factor (A) and its volume fraction in the FFRC are taken as 10 μm and 0.6, respectively. The diameter of the CFF (2R) and the length of straight CNT (L₀) are taken from Ray and Pradhan (2007); such that, the complex shear modulus, the Poisson’s ratio and the density of the viscoelastic layer are taken as 20(1 + i) MNm⁻², 0.49 and 1140 kg/m³, respectively (Chantalakhana and Stanway, 2001).

Table 1. Material properties of the base composite and the fuzzy fiber reinforced composite (FFRC) (Kundalwal and Ray, 2003; \( \nu_T = 0.6 \)).

<table>
<thead>
<tr>
<th>Composite material</th>
<th>( V_{\text{CNT}} ) and ( \omega )</th>
<th>( C_{11} ) (GPa)</th>
<th>( C_{12} ) (GPa)</th>
<th>( C_{13} ) (GPa)</th>
<th>( C_{22} ) (GPa)</th>
<th>( C_{23} ) (GPa)</th>
<th>( C_{44} ) (GPa)</th>
<th>( C_{55} ) (GPa)</th>
<th>( \rho ) (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base composite</td>
<td>( V_{\text{CNT}} )\text{max} = 0</td>
<td>144.98</td>
<td>7.89</td>
<td>7.89</td>
<td>14.31</td>
<td>8.63</td>
<td>2.84</td>
<td>3.44</td>
<td>1508</td>
</tr>
<tr>
<td>FFRC</td>
<td>( V_{\text{CNT}} = 0.0214 ), ( \omega = 0 )</td>
<td>145.86</td>
<td>9.02</td>
<td>9.02</td>
<td>23.95</td>
<td>12.11</td>
<td>5.92</td>
<td>4.05</td>
<td>1512</td>
</tr>
<tr>
<td>FFRC (2–3 plane)</td>
<td>( V_{\text{CNT}} = 0.0969 ), ( \omega = 32\pi/L_0 )</td>
<td>170.66</td>
<td>9.90</td>
<td>9.90</td>
<td>27.36</td>
<td>14.11</td>
<td>6.63</td>
<td>10.70</td>
<td>1528</td>
</tr>
<tr>
<td>FFRC (1–3 plane)</td>
<td>( V_{\text{CNT}} = 0.0969 ), ( \omega = 32\pi/L_0 )</td>
<td>258.22</td>
<td>12.62</td>
<td>12.62</td>
<td>26.90</td>
<td>13.03</td>
<td>6.48</td>
<td>10.90</td>
<td>1528</td>
</tr>
</tbody>
</table>

5.1. Comparisons with analytical results

To verify the validity of the present FE model, the natural frequencies of the simply supported (SS) symmetric and antisymmetric cross-ply, and antisymmetric angle-ply laminated composite plates integrated with the inactivated ACLD patches with negligible thickness are first computed and subsequently compared with the existing analytical results presented in Reddy (1996) for the identical laminated composite plates without integration of the ACLD patches. A non-dimensional frequency parameter \( \lambda \) is used for comparing the fundamental natural frequencies of the laminated composite plates, such that,

\[
\lambda = \bar{\omega} \left( \frac{a^2}{h} \right) \sqrt{\rho / E_T}
\]

in which \( \bar{\omega} \) represents the natural frequency of the overall plate; \( \rho \) and \( E_T \) are the density and the transverse Young’s modulus, respectively, of the orthotropic layers of the substrate composite plate. Table 2 summarizes the outcome of this comparison. It may be observed from this table that the results are in excellent agreement, validating the FE model derived in this study.

5.2. ACLD of laminated base composite and FFRC with straight CNT plates

Let us first investigate the damping characteristics of the laminated base composite (\( V_{\text{CNT}} = 0 \)) and FFRC (containing straight CNTs, \( \omega = 0 \)) plates integrated with the ACLD patches. For this, equations (27) and (28) are formulated to compute the frequency responses, while the plates are harmonically excited by a force of 1 N applied to a point (a/2, b/4, h/2). The control voltage applied to the first ACLD patch is negatively proportional to the velocity of a point (a/2, b/4, h/2) and that applied to the other ACLD patch is...
negatively proportional to the velocity of a point \((a/2, 3b/4, h/2)\). Figure 8 illustrates the frequency responses for the transverse displacement of a point \((a/2, b/4, h/2)\) of the laminated SS symmetric cross-ply \((0^\circ/90^\circ/0^\circ)\) base and FFRC plates. This figure displays both controlled and uncontrolled frequency responses, and clearly shows that the constraining layer made of the vertically reinforced 1–3 PZCs significantly attenuates the amplitude of vibrations and enhances the damping characteristics of the laminated plates over the passive damping (uncontrolled).

5.3. Effect of CNT waviness on the ACLD of the laminated FFRC plates

Let us now demonstrate the role played by the waviness of CNTs on the damping characteristics of the laminated symmetric/antisymmetric cross-ply and antisymmetric angle-ply FFRC plates. The effect of CNT waviness on the frequency responses of the SS symmetric cross-ply laminated FFRC plates is demonstrated in Figure 9. It may be observed from this figure that the damping characteristics of the laminated FFRC plates are improved when CNT waviness is coplanar with the 1–3 plane over those of the other laminated plates with and without CNTs. Although not presented here, the same is true for the SS antisymmetric cross-ply \((0^\circ/90^\circ/0^\circ)\) and antisymmetric angle-ply \((-45^\circ/45^\circ/-45^\circ/45^\circ)\) laminated FFRC plates. This is attributed to the fact that when the waviness of CNTs is coplanar with the 1–3 plane then the values of the effective elastic coefficients of the FFRC are significantly improved, which eventually enhances the attenuation of amplitude of vibrations and the natural frequencies of the laminated FFRC plates. The maximum control voltage required to achieve this attenuation is only 38 V when CNT waviness is coplanar with the 1–3 plane as shown in Figure 10. So far, in this work, the damping characteristics of the SS laminated FFRC plates have been studied. However, the imposition of clamped-clamped (CC) boundary conditions on the laminated FFRC plates may influence their damping characteristics. Therefore, the effect of CC boundary conditions on the damping characteristics of the laminated FFRC plates is also investigated here. The effect of CNT waviness on the frequency responses of the CC symmetric cross-ply laminated FFRC plates is illustrated in Figure 11 when the value of \(\psi\) is 0°. This figure depicts that the performance of the constraining layer made of the vertically reinforced 1–3 PZCs significantly improves the damping characteristics of the laminated symmetric/antisymmetric cross-ply and antisymmetric angle-ply FFRC plates.
laminated FFRC plates when CNT waviness is coplanar with the 1–3 plane over those of the laminated composite plates with and without CNTs. Unless otherwise mentioned, the value of control gain \(K_d\) is considered as 600 to investigate the effect of variation of piezoelectric fibers’ orientation on the performance of the ACLD patches for evaluating the subsequent results.

### 5.4. Effect of orientation angle of piezoelectric fibers on the ACLD of the laminated FFRC plates

The present work is further extended to study the effect of the orientation angle of piezoelectric fibers on the damping characteristics of laminated FFRC plates, because the performance of the ACLD treatment controlling the thin laminated composite structural elements is greatly affected by the piezoelectric fiber orientation angle in the two mutually orthogonal vertical planes (that is, xz and yz) of the PZC layer (Ray and Pradhan, 2007, 2008; Shah and Ray, 2012; Suresh Kumar and Ray, 2012). The maximum value of the orientation angle of piezoelectric fiber (\(\psi\)) in the commercially available obliquely reinforced 1–3 PZC is 45°. Hence, for studying the effect of piezoelectric fibers’ orientation on the performance of the ACLD patches, the value of \(\psi\) is varied from 0° to 45°. However, for the sake of clarity in the plots, the frequency responses corresponding to the four particular values of \(\psi\) (that is, 0°, 15°, 30° and 45°) have been presented in such a way that the optimum performance of the ACLD patches can be demonstrated. Figure 12 illustrates the effect of varying the orientation of piezoelectric fibers in the vertical xz plane on the performance of the ACLD patches for improving the damping characteristics of the SS symmetric cross-ply laminated plates when wavy carbon nanotubes are coplanar with the 1–3 plane.

![Figure 12. Effect of different values of \(\psi\) in the xz plane on the performance of the active constrained layer damping patches for controlling the simply supported symmetric cross-ply laminate plates with CNT waviness coplanar with the 1–3 plane.](image-url)

The same is true for the SS symmetric cross-ply laminated FFRC plates when CNT waviness is coplanar with the 2–3 plane while the orientation angle of piezoelectric fibers is varied in the vertical xz plane as depicted in Figure 13. Although not presented here, a similar effect is also observed for both cases of CNT waviness (that is, 1–3 plane and 2–3 plane) when the orientation angle...
of piezoelectric fibers in the PZC layer is varied in the yz plane. It may also be observed from Figures 12 and 13 that if CNT waviness is coplanar with the 1–3 plane then the natural frequencies of the laminated FFRC plates are improved over those containing wavy CNTs being coplanar with the 2–3 plane.

Figures 14 and 15 illustrate the damping characteristics of the SS antisymmetric cross-ply laminated FFRC plates when CNT waviness is coplanar with the 1–3 plane and the 2–3 plane, respectively, considering the variation of piezoelectric fibers’ orientation in the vertical xz plane. Once again, it is observed that the natural frequencies of the laminated FFRC plates containing wavy CNTs being coplanar with the 1–3 plane are enhanced over those containing wavy CNTs being coplanar with the 2–3 plane. Also, the attenuating capability of the ACLD patches becomes maximum when the value of $\psi$ in the xz plane is 0°. Similar results are also obtained for both cases of CNT waviness when the orientation angle of the piezoelectric fibers is varied in the yz plane, but these results are not presented here. Although they are not presented, it is also revealed that the attenuating capability of the ACLD patches becomes maximum when the value $\psi$ in the vertical xz and yz planes is 0° for the SS antisymmetric angle-ply laminated FFRC plates irrespective of the plane of wavy CNTs. But the significant improvement has been observed in the damping characteristics of the CC symmetric cross-ply laminated FFRC plates when CNT waviness is coplanar with the 1–3 plane. Since wavy CNTs coplanar with the 1–3 plane significantly improve the damping characteristics of the laminated FFRC plates, the effect of variation of piezoelectric fibers orientation on the damping characteristics of the CC laminated FFRC plates has been studied considering wavy CNTs being coplanar with the 1–3 plane.

Figures 16 and 17 illustrate the damping characteristics of the CC symmetric cross-ply laminated FFRC plates considering the variation of piezoelectric fibers’ orientation in the vertical xz and yz planes, respectively. These figures reveal that the damping characteristics of the CC symmetric cross-ply laminated FFRC plates are
improved when the values of $\psi$ in the vertical xz and yz planes are $30^\circ$ and $15^\circ$, respectively. The same is true for the CC antisymmetric angle-ply laminated FFRC plates when the orientation angle of piezoelectric fibers is varied in the vertical xz and yz planes as depicted in Figures 18 and 19, respectively.

Figures 20 and 21 reveal that the performance of the ACLD patches becomes maximum for controlling the CC antisymmetric cross-ply ($0^\circ/90^\circ/0^\circ$) fuzzy fiber reinforced composite plates when wavy carbon nanotubes are coplanar with the 1–3 plane.

is $30^\circ$. Although not presented here, similar effects of variation of piezoelectric fibers’ orientation for improving the damping characteristics of the laminated symmetric/antisymmetric cross-ply and antisymmetric angle-ply CC laminated FFRC plates were observed when CNT waviness is coplanar with the 2–3 plane, but the improvement is not as significant as that observed in the case of CNT waviness coplanar with the 1–3 plane.
6. Conclusions

In this paper, a study has been carried out to investigate the effect of CNT waviness on the damping characteristics of the smart laminated FFRC plates. The distinctive feature of the construction of a novel FFRC is that CNTs are radially grown on the circumferential surfaces of carbon fibers. An FE model has been developed to describe the dynamics of the laminated composite plates integrated with the ACLD patches. The following main conclusions are drawn from the investigations carried out in this paper.

1. Since the radially grown CNTs on the circumferential surfaces of carbon fibers eventually stiffen the polymer matrix in the radial directions, the transverse effective elastic properties of the resulting composite improve, which causes the attenuation of amplitude of vibrations and the natural frequencies of the laminated FFRC plates to be enhanced over those of the laminated base composite plates (that is, without CNTs).

2. The damping characteristics of the symmetric/antisymmetric cross-ply and antisymmetric angle-ply laminated FFRC plates containing wavy CNTs being coplanar with the 1–3 plane are improved over those containing straight CNTs or wavy CNTs being coplanar with the 2–3 plane irrespective of the imposition of different boundary conditions on the laminated FFRC plates.

3. The performance of the ACLD patches becomes maximum for controlling the SS symmetric/antisymmetric cross-ply and antisymmetric angle-ply laminated FFRC plates when the orientation angle of piezoelectric fibers in the vertical xz and yz planes is 0° irrespective of the plane of CNT waviness.

4. For the CC symmetric cross-ply and antisymmetric angle-ply laminated FFRC plates the performance of the ACLD patches becomes maximum when the orientation angles of piezoelectric fibers in the vertical xz and yz planes are 30° and 15°, respectively, whereas for the CC antisymmetric cross-ply laminated FFRC plates the performance of the ACLD patches becomes maximum when the orientation angle of piezoelectric fibers in the vertical xz and yz planes is 30° irrespective of the plane of CNT waviness.

Based on the above points it may be concluded that wavy CNTs can be utilized effectively to improve the damping behavior of multifunctional nano-tailored composite structures exploiting their remarkable direction-oriented elastic properties.

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Nomenclature

Abbreviations

ACLD Active constrained layer damping
CC Clamped-clamped
CFF Composite fuzzy fiber
CNT  | Carbon nanotube
FE  | Finite element
FFRC | Fuzzy fiber reinforced composite
PZC  | Piezoelectric composite
SEM  | Scanning electron microscopy
SS  | Simply supported
RVE  | Representative volume element

**Notations**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Amplitude of the CNT (m)</td>
</tr>
<tr>
<td>a, b</td>
<td>Length and width of the substrate plate (m)</td>
</tr>
<tr>
<td>$C_{ij}$</td>
<td>Elastic coefficients of the kth layer (GPa)</td>
</tr>
<tr>
<td>$[C_{ij}], [C_{ij}]$</td>
<td>Elastic coefficient matrices for the kth layer (GPa)</td>
</tr>
<tr>
<td>$D_{xx}, D_{yy}, D_{zz}$</td>
<td>Electric displacements in the x, y and z directions, respectively (C/m²)</td>
</tr>
<tr>
<td>$[D_{hs}], [D_{hs}]$</td>
<td>Rigidity matrices for the kth layer $[D_{hs}], [D_{hs}]$</td>
</tr>
<tr>
<td>$d_n$</td>
<td>Diameter of the CNT (m)</td>
</tr>
<tr>
<td>${d_i}, {d_i}$</td>
<td>Generalized translational and rotational displacements</td>
</tr>
<tr>
<td>${d_i^e}, {d_i^e}$</td>
<td>Nodal generalized translational and rotational displacements</td>
</tr>
<tr>
<td>E</td>
<td>Young’s modulus of the viscoelastic material (GPa)</td>
</tr>
<tr>
<td>$E_T$</td>
<td>Transverse Young’s modulus of the kth layer (GPa)</td>
</tr>
<tr>
<td>$E_x, E_y, E_z$</td>
<td>Applied electric field components in the x, y and z directions, respectively (C/m or V/m)</td>
</tr>
<tr>
<td>$[\epsilon_{hs}], [\epsilon_{hs}]$</td>
<td>Piezoelectric stress coefficient vectors (C/m²)</td>
</tr>
<tr>
<td>$\tilde{\epsilon}_{ij}$</td>
<td>Transformed effective piezoelectric coefficients of the PZC layer (C/m²)</td>
</tr>
<tr>
<td>${F}$</td>
<td>Global nodal load vector (N)</td>
</tr>
<tr>
<td>${F_{lp}}, {F_{lp}}$</td>
<td>Global electro-elastic coupling vectors (V/m)</td>
</tr>
<tr>
<td>${F_e}, {F_e}$</td>
<td>Nodal load vector (N)</td>
</tr>
<tr>
<td>${\tilde{F}_e}, {\tilde{F}_e}$</td>
<td>Elemental electro-elastic coupling vectors (C/m)</td>
</tr>
<tr>
<td>G</td>
<td>Complex shear modulus of the viscoelastic material (GPa)</td>
</tr>
<tr>
<td>$h_1, h_2$</td>
<td>z-coordinates of the top surfaces of the viscoelastic and PZC layers (m)</td>
</tr>
<tr>
<td>h, h_v, h_p</td>
<td>Thicknesses of the substrate plate, viscoelastic layer and PZC layer, respectively (m)</td>
</tr>
<tr>
<td>$K_d$</td>
<td>Control gain</td>
</tr>
<tr>
<td>$[K_{ii}], [K_{ii}]$</td>
<td>Global stiffness matrices (N/m)</td>
</tr>
<tr>
<td>$[K_{ii}^e], [K_{ii}^e]$</td>
<td>Elemental stiffness matrices (N/m)</td>
</tr>
<tr>
<td>L</td>
<td>Half length of the RVE of the FFRC (m)</td>
</tr>
<tr>
<td>$L_n$</td>
<td>Length of straight CNT (m)</td>
</tr>
<tr>
<td>$L_{nr}$</td>
<td>Running length of sinusoidaly wavy CNT (m)</td>
</tr>
<tr>
<td>$[M], [M^e]$</td>
<td>Global and elemental mass matrices (kg)</td>
</tr>
<tr>
<td>N</td>
<td>Total number of layers in the substrate plate</td>
</tr>
<tr>
<td>$[N_i], [N_i]$</td>
<td>Shape function matrices</td>
</tr>
<tr>
<td>$n_i$</td>
<td>Shape function of natural coordinates associated with ith node</td>
</tr>
<tr>
<td>q</td>
<td>Number of patches of the ACLD treatment</td>
</tr>
<tr>
<td>R</td>
<td>Radius of the RVE of the composite fuzzy fiber (m)</td>
</tr>
<tr>
<td>$u, v, w$</td>
<td>Displacements along the x, y and z directions, respectively (m)</td>
</tr>
<tr>
<td>$u_0, v_0, w_0$</td>
<td>Displacements of a point on the reference mid-plane along the x, y and z directions, respectively (m)</td>
</tr>
<tr>
<td>V</td>
<td>Electric potential (voltage)</td>
</tr>
<tr>
<td>$V_{CNT_{max}}$</td>
<td>Maximum CNT volume fraction in the FFRC</td>
</tr>
<tr>
<td>$v_r$</td>
<td>Carbon fiber volume fraction in the FFRC</td>
</tr>
<tr>
<td>$x, y, z$</td>
<td>Cartesian coordinates</td>
</tr>
<tr>
<td>$[X], [X_r]$</td>
<td>Global nodal translational and rotational displacement vectors (m)</td>
</tr>
<tr>
<td>$\gamma_x, \gamma_y, \gamma_z$</td>
<td>Generalized rotations of the normal to the middle plane of the PZC layer (rad)</td>
</tr>
<tr>
<td>${\epsilon_h}, {\epsilon_s}$</td>
<td>In-plane and transverse strain vectors</td>
</tr>
</tbody>
</table>
References


Moradi-Dastjerdi R, Pourashgar A and Foroutan M (2013) The effects of carbon nanotube orientation and aggregation on vibrational behavior of functionally graded...


Appendix A

In equations (6) and (7), the matrices $[Z_1]$, $[Z_2]$, $[Z_3]$, $[Z_4]$ and $[Z_5]$ are given by

$$
[Z_1] = \begin{bmatrix} Z_1 & \tilde{Z}_1 \\ \tilde{Z}_2 & Z_2 \end{bmatrix}, \\
[Z_2] = \begin{bmatrix} (h/2)I & \tilde{Z}_2 \\ \tilde{Z}_2 & (h/2)I \end{bmatrix}, \\
[Z_3] = \begin{bmatrix} \tilde{Z}_3 & \tilde{Z}_3 \\ \tilde{Z}_3 & \tilde{Z}_3 \end{bmatrix}, \\
[Z_4] = \begin{bmatrix} \tilde{Z}_4 & \tilde{Z}_4 \\ \tilde{Z}_4 & \tilde{Z}_4 \end{bmatrix}, \\
[Z_5] = \begin{bmatrix} \tilde{Z}_5 & \tilde{Z}_5 \\ \tilde{Z}_5 & \tilde{Z}_5 \end{bmatrix}
$$

and

$$
[Z_6] = \begin{bmatrix} \tilde{Z}_6 & \tilde{Z}_6 \\ \tilde{Z}_6 & \tilde{Z}_6 \end{bmatrix}
$$

in which

$$
\begin{align*}
[Z_1] &= \begin{bmatrix} z & 0 & 0 & 0 \\ 0 & z & 0 & 0 \\ 0 & 0 & z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, & [\tilde{Z}_2] &= \begin{bmatrix} (z-h/2) & 0 & 0 & 0 \\ 0 & (z-h/2) & 0 & 0 \\ 0 & 0 & (z-h/2) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \\
[Z_3] &= \begin{bmatrix} (z-h_{N+1}) & 0 & 0 & 0 \\ 0 & (z-h_{N+1}) & 0 & 0 \\ 0 & 0 & (z-h_{N+1}) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, & [I] &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.
\end{align*}
$$

The various sub-matrices $B_{bi}$, $B_{ti}$, $B_{rbi}$ and $B_{rsi}$ appearing in equation (24) are given by

$$
\begin{align*}
B_{bi} &= \begin{bmatrix} \frac{\partial n_i}{\partial x} & 0 & 0 \\ 0 & \frac{\partial n_i}{\partial y} & 0 \\ \frac{\partial n_i}{\partial y} & \frac{\partial n_i}{\partial x} & 0 \\ 0 & 0 & 0 \end{bmatrix}, & B_{tsi} &= \begin{bmatrix} 0 & 0 & \frac{\partial n_i}{\partial x} \\ 0 & 0 & \frac{\partial n_i}{\partial y} \end{bmatrix}, \\
B_{rbi} &= \begin{bmatrix} \tilde{B}_{rbi} & 0 & 0 \\ 0 & \tilde{B}_{rbi} & 0 \\ 0 & 0 & \tilde{B}_{rbi} \end{bmatrix}, & \tilde{B}_{rbi} &= \begin{bmatrix} \frac{\partial n_i}{\partial x} & 0 & 0 \\ 0 & \frac{\partial n_i}{\partial y} & 0 \\ \frac{\partial n_i}{\partial y} & \frac{\partial n_i}{\partial x} & 0 \end{bmatrix}, \\
B_{rsi} &= \begin{bmatrix} \tilde{B}_{rsi} & 0 & 0 \\ 0 & \tilde{B}_{rsi} & 0 \\ 0 & 0 & \tilde{B}_{rsi} \end{bmatrix}, & \tilde{B}_{rsi} &= \begin{bmatrix} 0 & 0 & \frac{\partial n_i}{\partial x} \\ 0 & 0 & \frac{\partial n_i}{\partial y} \end{bmatrix},
\end{align*}
$$

$$
\tilde{\phi} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \tilde{\phi} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad \tilde{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.
$$