Dynamic modelling and analysis of smart carbon nanotube-based hybrid composite beams: Analytical and finite element study

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Abstract
In this work, the carbon nanotube-based hybrid carbon fibre-reinforced composite smart beam constraining the layer of an active constrained layer damping treatment is investigated using an in-house finite element model based on first-order shear deformation theory. The effect of in-plane and transverse-plane actuation of the integrated active constrained layer damping treatment layer on the damping characteristics of the novel smart cantilever hybrid carbon fibre-reinforced composite beam is considered. The parameters affecting the damping characteristics of the hybrid carbon fibre-reinforced composite substrate beam such as the volume fraction of both carbon nanotubes and carbon fibre, and the aspect ratio are also studied. Besides, the micromechanical model based on the mechanics of materials approach is developed to estimate the effective elastic coefficient of novel hybrid carbon fibre-reinforced composite laminate. The effective properties of hybrid carbon fibre-reinforced composite are predicted quantitatively by considering non-bonded interaction formed between carbon nanotubes and the polymer matrix. It is revealed that due to the incorporation of carbon nanotubes into the epoxy matrix, the effective longitudinal, transverse and shear properties of the hybrid carbon fibre-reinforced composite lamina are significantly enhanced. Our outcomes explore that the damping performance of the laminated hybrid carbon fibre-reinforced composite smart beam considering the incorporation of carbon nanotubes shows substantial improvement as compared to the base composite. To bring more clarity, the quantitative relative performance of hybrid carbon fibre-reinforced composite and base composite is presented. Our fundamental analysis sheds the light on the opportunities of developing efficient, high-performance and lightweight carbon nanotubes-based micro-electro-mechanical systems smart structures such as sensors, actuators and distributors.

Keywords
Active damping, carbon nanotubes, hybrid composite, 1-3 piezoelectric composite, finite element model, micromechanical model

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Introduction
In the last two decades, polymer composites captivated a great interest of the researcher’s community due to their excellent thermo-mechanical properties.1–5 To meet the increasing demands of high mechanical strength and energy-efficient material, several researchers developed carbon nanotubes (CNTs)-reinforced polymer composites. Due to the extraordinary thermo-mechanical properties, these CNT-based composite structures such as beams, plates and shells have a wide range of applications in vibration attenuation,6–8 noise control9,10 and structural health monitoring (SHM) systems.11,12 The study of such laminated hybrid composite structures integrated with conventional piezoelectric, laminated piezoelectric and piezoelectric composite actuators and sensors have drawn the attention of researchers from fundamental and application point of view. The emphasis has been given on evolving the MEMS, which may be utilized for developing smart
energy harvesters. The advancement in this research area can lead to the development of lightweight, high strength, high stiffness and efficient smart engineering structures with self-controlling and monitoring capabilities.\textsuperscript{13–17}

Piezoelectricity was first discovered by French scientists, Jacques and Pierre Curie in 1880. Soon after, Gabriel Lippmann determined a mathematical relation for the converse piezoelectric effect from the fundamental principles of thermodynamics in 1881, which was not estimated by Curie brothers. The piezoelectric phenomenon is well known for generating an electrical response when subjected to uniform mechanical deformation, known as the direct piezoelectric effect. Whereas a mechanical deformation occurs when subjected to electrical field, known as the converse piezoelectric effect. Such electromechanical coupling presents in non-centrosymmetric crystals. The inversion centre is not present in such non-centrosymmetric crystalline materials, which in-results generate polarization when it is subjected to mechanical deformation. Hence, piezoelectric materials can be used as a suitable candidate in the applications of MEMS such as sensors and actuators in the form of beams, plates and shells. For instance, Nazargah and Reddy extensively studied the dynamic behaviour of a functionally graded piezoelectric bimorph and sandwich composite beam structures using the commonly available piezoelectric materials such as lead zirconate titanate (PZT)-5H and polyvinylidene fluoride.\textsuperscript{14,18–21} Kundalwal et al.\textsuperscript{13,22} examined the linear active damping of a smart fuzzy fibre-reinforced composites (FFRC) attached with active constrained layer damping (ACLD) treatment layer constraining 1–3 piezoelectric composite (PZC) material. They showed the enhancement in the active damping property of the smart structures, which caused a reduction in respective vibrations. Most recently, a graphene-based PZC was also being studied by Shingare et al.\textsuperscript{23–26} They predicted electromechanical properties of the graphene-based nanocomposites with micromechanical model based on the mechanics of materials (MOM),\textsuperscript{37} modified strength of materials and finite element (FE) approach. Their results showed enhanced out-of-plane piezoelectric constant \( (\varepsilon_{33}) \) as compared to in-plane piezoelectric constant \( (\varepsilon_{31}) \); the similar trend was also observed by Ray and Batra team.\textsuperscript{28–31} The effect of non-uniform imperfect contact between fibre and matrix phase was investigated by several researchers. It should be noted that the interface conditions has significant impact on the elastic properties of composites at higher fibre volume fraction.\textsuperscript{32–34}

The CNTs can be utilized as nanofillers in the conventional matrices due to their high electro-thermo-mechanical properties to improve the properties of the composites. The helical microtubules of graphitic carbon, CNTs have drawn significant attention among researchers since 1991 to predict its thermo-electro-mechanical properties.\textsuperscript{35} Based on the experimental analysis, in the year 1996, Treacy et al.\textsuperscript{36} found the extraordinarily high Young’s modulus of CNTs ranging in tera-pascal (TPa), which leads the investigators to carry out theoretical models to evaluate the mechanical properties of CNTs.\textsuperscript{37–39} Numerically, Sears and Batra predicted the axial Young’s modulus of the single-walled carbon nanotubes (SWCNT), and they also revealed that the axial Young’s modulus and the thickness of SWCNT do not depend on chirality.\textsuperscript{40} Most recently, Kundalwal and Choyal\textsuperscript{41} studied the vacancy defect in CNT and evaluated transversely isotropic elastic properties of the defected CNT through molecular dynamics simulation. They also observed that such vacancies have more influence on in-plane shear moduli but have very little effects on the axial Young’s modulus of defective CNT. Owing to such extraordinary properties of CNT, it is extensively used as a nanofiller, in a surge to develop the lightweight and the high-performance composites.

In recent advances, CNTs-based composite structures were frequently used in numerous MEMS applications. It was observed that the performance of the composite materials enhances significantly by adding a small number of CNTs. For an instance, Chen and Liu explored that the stiffness of the composite materials was improved by 33% in an axial direction by incorporating 3.6% of CNT volume fraction in the matrix material.\textsuperscript{42} Using the ball milling technique, Esawi et al. showed the enhancement of tensile strength by 50% and stiffness by 23% due to the addition of 5 wt.% of CNT in the aluminum matrix.\textsuperscript{43} Through an experimental investigation such as open hole tension, shear beam test and flatwise tension tests, Tarfaoui et al. found a significant control in crack propagation by adding CNT up to 2% volume fraction in the polymer matrix, which improved the overall thermo-elastic properties of the composites.\textsuperscript{44} Similar results were observed by Park et al.\textsuperscript{45} and they observed the highest Young’s modulus and mechanical properties at 2% volume fraction of CNT in the epoxy matrix. From the above literature, researchers observed that CNTs tends to agglomerate when reinforced with matrix phase and hence weaken the junction, which in result limits use of CNTs as a nanofiller in the composite materials up to a certain wt.%. Further research was carried out to increase the wt.% of CNT in the matrix phase by developing the new techniques.\textsuperscript{46–48} By using the novel shear pressing method, several researchers reported that the high-volume fraction CNT-based hybrid composites are possible when the conventional micro-fibres were used. Bradford et al.\textsuperscript{49} confirmed 27% volume fraction of the long aligned CNTs (in order of mm) in the composite materials with suitable microfibres. Huang et al. reported 20% multiwalled carbon nanotubes (MWCNTs) along with 20% graphene nanoplatelet in an epoxy matrix hybrid
composite through a well-designed fabrication method. Ultra-high wt. percent of MWCNT (68 wt.%) was reported by Mecklenburg et al. in the epoxy-based composite using the novel hot-press infiltration manufacturing process through a semi-permeable membrane with long aligned MWCNTs (in order of mm) grown using chemical vapour deposition process. Most recently, Rathi and Kundalwal developed an epoxy-based hybrid composites with MWCNTs as a fibre and ZrO$_2$ as an interface using novel ultrasonic dual mixing process with CNTs dispersion of ~6% by wt. Therefore, the present research aims to further study the importance of the presence of nanofillers into the pure matrix to form CNTs-based hybrid composites.

At this juncture, the CNT-based hybrid composites are extensively studied in the application of vibration-damping, as CNTs possess excellent damping coefficient because of their high strength to weight ratio and stiffness properties. Ray and Batra reported the CNT-based 1–3 piezoelectric composites (NRPEC) compelled with the layer of ACLD through the FE approach, and they observed the better damping results using NRPEC when compared with conventionally available PZC. Using an analytical approach, Lin and Lu studied the damping behaviour of CNT-based composites, and they also showed that a small volume fraction of 1% CNT can result in 20% loss in the damping factor. Kundalwal and Meguid derived the FE model for the active damping to investigate the influence of waviness of CNTs on the FFRC shell. They observed that the damping characteristics of FFRC composite shells were significantly influenced by CNT waviness.

The review of the above literature on the CNTs-based composite specifies that the CNT is one of the most fascinating nanofiller material immensely studied in the past few decades. Surprisingly, the application of effective properties of transversely isotropic CNTs for modelling of CNTs-based structures in conjunction with conventional piezoelectric material are not explored yet, and this has motivated the current work. The objective of this research work is to further extend the understanding of the presence of nanofillers in the composite to form the various MEMS smart structures such as smart beams. It was found that the beams and cantilevers are two significant types of MEMS structures. Cantilever composite beam structures possess more potential due to their high sensitivity as well as linear elastic behaviour. Therefore, the rationale of the present study is to investigate the damping characteristics of a composite due to the utilization of CNTs as nanofillers. We intend to explore the performance of hybrid carbon fibre-reinforced composite (HFRC) in the active damping of smart composite beam constraining ACLD treatment using an in-house developed FE approach. The present article is organized as follows: The upcoming section presents a micromechanical model based on a two-phase and three-phase MOM approach to estimate the effective elastic properties of base composite and HFRC lamina, respectively. In a further section, we present a FE model for the smart HFRC cantilever beam, to investigate its overall damping performance. A closed-loop model is also presented to calculate the required control voltage for the active damping. In the final section, the results and discussions are presented with quantitative analysis.

**Effective elastic properties of HFRC**

In this section, the effective elastic properties of the HFRC composite were estimated by using the two- and three-phase micromechanical model based on the MOM approach. A novel HFRC was composed of carbon fibre and epoxy matrix by considering the rectangular representative volume elements (RVEs) incorporated with cylindrical fibres. In this micromechanical analysis, we restricted ourself to a single RVE. Figure 1(a) presents the schematic of HFRC lamina reinforced with carbon fibre in one-axis and epoxy matrix mixed with CNT to improve damping and material properties of the matrix. The axial and transverse cross-sections of base composite and HFRC RVE is illustrated in Figure 1(b) and (c), respectively.

The principal material coordinate (x-y-z) or problem coordinate (1-2-3) system was followed for deriving the MOM model as both coordinate systems exactly matched with each other. The stresses and strains developed in the HFRC were illustrated based on this coordinate system, where one-axis is known as the fibre axis, while the other two axes (i.e., two- and three-axis) are known as the matrix axis. According to Hooke’s law, the constitutive relation for the individual phases of HFRC can be expressed as follows

$$\{\sigma\} = [C_r]\{\varepsilon\}; \ r = CF \ and \ Exy \quad (1)$$

where the stress–strain vector and stiffness matrix are represented by $\{\sigma\}$, $\{\varepsilon\}$ and $[C]$ for rth phase,
respectively. The superscript CF and Exy denote carbon fibre and epoxy matrix, respectively, while no superscript is used for composite lamina. In equation (2), $\sigma_1$, $\sigma_2$ and $\sigma_3$ represent the normal stresses; $\varepsilon_1$, $\varepsilon_2$ and $\varepsilon_3$ represent the normal strains in the respective 1, 2 and 3 directions; $\sigma_{12}$, $\sigma_{13}$ and $\sigma_{23}$ represent shear stresses; $\varepsilon_{12}$, $\varepsilon_{13}$ and $\varepsilon_{23}$ represent shear strains and $C_{ij}$ indicates the elastic stiffness coefficients.

Using iso-field (iso-stress and iso-strain) and rules-of-mixture (ROM) conditions, the following assumptions are considered:

- The analysis is linearly elastic.
- Composite is homogeneous throughout.
- Fibres and nanofillers are aligned in one-axis, i.e. axial direction.
- Fibres and nanofillers are parallel and continuous.
- Fibre, nanofiller and matrix are equally long.
- No slippage between fibre, nanofiller and matrix.

Two-phase MOM approach

The two-phase MOM model is developed by modifying the MOM model derived by Kundalwal and Ray to determine the transversely isotropic effective elastic properties of a base composite constituting two phases such as carbon fibre and epoxy matrix. To satisfy no slippage condition between fibre and matrix.

Figure 1. (a) Schematic diagram of HFRC lamina; axial and transverse cross-sections of HFRC RVE considering: (b) two-phase and (c) three-phase.
matrix, the assumption of iso-field and ROM can be expressed as

In the case of iso-field condition

\[
\begin{pmatrix}
\sigma_{11}^{CF} \\
\sigma_{22}^{CF} \\
\sigma_{33}^{CF} \\
\sigma_{12}^{CF} \\
\sigma_{13}^{CF} \\
\sigma_{23}^{CF}
\end{pmatrix}
+ v_{Exy}
\begin{pmatrix}
\sigma_{11}^{Exy} \\
\sigma_{22}^{Exy} \\
\sigma_{33}^{Exy} \\
\sigma_{12}^{Exy} \\
\sigma_{13}^{Exy} \\
\sigma_{23}^{Exy}
\end{pmatrix}
= \begin{pmatrix}
\sigma_{11} \\
\sigma_{22} \\
\sigma_{33} \\
\sigma_{12} \\
\sigma_{13} \\
\sigma_{23}
\end{pmatrix}
\] (3)

and in case of ROM condition

\[
\begin{pmatrix}
\sigma_{11}^{CF} \\
\sigma_{22}^{CF} \\
\sigma_{33}^{CF} \\
\sigma_{12}^{CF} \\
\sigma_{13}^{CF} \\
\sigma_{23}^{CF}
\end{pmatrix}
+ v_{Exy}
\begin{pmatrix}
\sigma_{11}^{Exy} \\
\sigma_{22}^{Exy} \\
\sigma_{33}^{Exy} \\
\sigma_{12}^{Exy} \\
\sigma_{13}^{Exy} \\
\sigma_{23}^{Exy}
\end{pmatrix}
= \begin{pmatrix}
\sigma_{11}^{CF} \\
\sigma_{22}^{CF} \\
\sigma_{33}^{CF} \\
\sigma_{12}^{CF} \\
\sigma_{13}^{CF} \\
\sigma_{23}^{CF}
\end{pmatrix}
+ v_{Exy}
\begin{pmatrix}
\sigma_{11}^{Exy} \\
\sigma_{22}^{Exy} \\
\sigma_{33}^{Exy} \\
\sigma_{12}^{Exy} \\
\sigma_{13}^{Exy} \\
\sigma_{23}^{Exy}
\end{pmatrix}
\] (4)

where \( v_{CF} \) and \( v_{Exy} \) denote the volume fraction of carbon fibre and epoxy matrix and \( v_{Exy} = 1 - v_{CF} \). By using equations (1), (3) and (4), the stress and strain vector in the base composite can be written with respect to their constituent phases. The relations obtained in the development of two-phase model are explicitly written in Supplementary file section S1.

Finally, the effective elastic properties of the two-phase MOM model can be expressed as

\[
[C] = [C_1][V_3]^{-1} + [C_2][V_4]^{-1}
\] (5)

\[
[V_3] = [V_1] + [V_2][C_4]^{-1}[C_3]
\] (6)

\[
[V_4] = [V_2] + [V_1][C_3]^{-1}[C_4]
\]

Three-phase MOM approach

To enhance the optimal performance of a composite, there should be an interaction between fibre and matrix. This could be done by surface treating the fibre or matrix or both before using them. The surface treatment of fibre is the most commonly used technique called sizing. Such treatment on the matrix to improve its adhesive property due to the addition of nanofillers is promising and received a lot of attention from the researchers.\(^5\)\(^2\),\(^6\)\(^0\) In this study, we are treating the epoxy matrix before impregnating carbon fibre into it, as shown in Figure 1(c). For the three-phase composite material which is also known as a hybrid composite material, the constitutive relation for the individual phases of HFRC can be expressed as

\[
\{\sigma^r\} = [C^r]\{\varepsilon^r\}; \quad r = \text{CF, CNT and Exy}
\] (7)

where the superscripts CF, CNT and Exy denote carbon fibre, CNT nanofiller and epoxy matrix, respectively. To satisfy no slippage condition between all individual phases, the assumption of iso-field and ROM can be expressed as

In the case of iso-field condition

\[
\begin{pmatrix}
\sigma_{11}^{CF} \\
\sigma_{22}^{CF} \\
\sigma_{33}^{CF} \\
\sigma_{12}^{CF} \\
\sigma_{13}^{CF} \\
\sigma_{23}^{CF}
\end{pmatrix}
+ v_{Exy}
\begin{pmatrix}
\sigma_{11}^{Exy} \\
\sigma_{22}^{Exy} \\
\sigma_{33}^{Exy} \\
\sigma_{12}^{Exy} \\
\sigma_{13}^{Exy} \\
\sigma_{23}^{Exy}
\end{pmatrix}
= \begin{pmatrix}
\sigma_{11}^{CF} \\
\sigma_{22}^{CF} \\
\sigma_{33}^{CF} \\
\sigma_{12}^{CF} \\
\sigma_{13}^{CF} \\
\sigma_{23}^{CF}
\end{pmatrix}
+ v_{Exy}
\begin{pmatrix}
\sigma_{11}^{Exy} \\
\sigma_{22}^{Exy} \\
\sigma_{33}^{Exy} \\
\sigma_{12}^{Exy} \\
\sigma_{13}^{Exy} \\
\sigma_{23}^{Exy}
\end{pmatrix}
\] (8)

and in case of ROM condition

\[
\begin{pmatrix}
\sigma_{11}^{CF} \\
\sigma_{22}^{CF} \\
\sigma_{33}^{CF} \\
\sigma_{12}^{CF} \\
\sigma_{13}^{CF} \\
\sigma_{23}^{CF}
\end{pmatrix}
+ v_{Exy}
\begin{pmatrix}
\sigma_{11}^{Exy} \\
\sigma_{22}^{Exy} \\
\sigma_{33}^{Exy} \\
\sigma_{12}^{Exy} \\
\sigma_{13}^{Exy} \\
\sigma_{23}^{Exy}
\end{pmatrix}
= \begin{pmatrix}
\sigma_{11}^{CF} \\
\sigma_{22}^{CF} \\
\sigma_{33}^{CF} \\
\sigma_{12}^{CF} \\
\sigma_{13}^{CF} \\
\sigma_{23}^{CF}
\end{pmatrix}
+ v_{Exy}
\begin{pmatrix}
\sigma_{11}^{Exy} \\
\sigma_{22}^{Exy} \\
\sigma_{33}^{Exy} \\
\sigma_{12}^{Exy} \\
\sigma_{13}^{Exy} \\
\sigma_{23}^{Exy}
\end{pmatrix}
\] (9)

where \( v_{CNT} \) denotes the volume fraction of the CNT. Finally, the effective elastic properties of HFRC can be expressed as

\[
[C] = [C_1][V_3]^{-1} + [C_7][V_6]^{-1}
\] (10)

The relations obtained in the development of three-phase model are explicitly written in Supplementary file section S2.

FE modelling of a smart beam

The FE model is derived to investigate the performance of the novel laminated HFRC smart beam attached with ACLD treatment constraining layer of 1–3 PZC material at upper surface of the beam, comprising \( N \) numbers of HFRC lamina as shown in Figure 2. All the layers of the HFRC beam are assumed transversely isotropic, uniformly homogeneous and linearly elastic, and the ACLD layer is composed of viscoelastic material.

The volume of the HFRC substrate beam is determined using a simple relation: \( L \times b \times h \), where \( L \), \( b \) and \( h \) denote the respective length, width and height of the beam, respectively. While \( L_o \) denotes the length of
ACLD treatment constraining layer of piezo material; \( h_c \) and \( h_p \) denote the respective thickness of ACLD and the 1–3 PZC layer, respectively. The reference plane of the laminated HFRC smart beam was considered as the mid-plane of the substrate beam. The global coordinate system was defined in such a way that its origin is situated on the reference plane. The smart beam is subjected to cantilevered boundary conditions (\( x = 0 \), and \( L \)). Here, the first-order shear deformation theory (FSDT) is used for modelling the axial displacement in each layer of an overall beam which can be considered as a thin beam. According to the FSDTs, Figure 3 illustrates the kinematics of deformation of the laminated HFRC smart beam in the axial direction. Here, the generalized translational displacement at any point on the reference plane (\( z = 0 \)) is denoted by \( u_0 \). \( \theta_x \), \( \phi_z \) and \( \gamma_z \) represent the generalized rotations of the normals to the mid planes of the HFRC substrate beam, the ACLD and the 1–3 PZC, respectively, in the \( xz \) plane. The coordinate \( z \) in the thickness direction of the upper and lower surface of any \((k - th)\) layer of the overall composite beam is given by \( h_{k+1} \) and \( h_k \) (\( k = 1, 2, 3, k1N + 2 \)), respectively, as shown in Figure 3. According to the FSDT, the axial displacement \( u \) at any point of the beam in the \( x \)-direction can be written as

\[
\begin{align*}
\mathbf{u}(x,z,t) &= u_0(x,t) + (z - (z - h/2))\theta_x(x,t) \\
&+ ((z - h/2) - (z - h_{N+2}))\phi_z(x,t) \\
&+ (z - h_{N+2})\gamma_z(x,t)
\end{align*}
\]  

(11)

where the function within brackets \( \langle \cdot \rangle \) denotes suitable singularity function which is also termed as zigzag function. In a general layer-wise formulation of a beam theory, for the modelling of axial deformation, the zigzag beam theory is used as shown in equation (11). The axial displacement field could be modelled as a piecewise continuous function that is a collection of linear functions defined for each layer. This theory consists of three such piecewise continuous expressions that accounts for the displacements in the HFRC substrate beam, ACLD layer and the 1–3 PZC layer. The proposed zigzag function vanishes at the top and bottom surfaces of the beam and does not require full shear-stress continuity across the thickness of laminated beam. Also, this theory appears as a natural extension to the FSDT for laminated-composite beams. We considered the transverse normal strain in the model, as to control the transverse amplitudes of deflection the transverse actuation is used. Due to the consideration of thin beam analysis, the variation of the transverse displacement \( (w) \) in the thickness direction at any point of the HFRC base beam, ACLD and constraining layer are considered to be affine across the thickness direction. Hence, in the case of the overall beam, the transverse displacement can be expressed as

\[
\begin{align*}
\mathbf{w}(x,z,t) &= w_0(x,t) + (z - (z - h/2))\theta_z(x,t) \\
&+ ((z - h/2) - (z - h_{N+2}))\phi_z(x,t) \\
&+ (z - h_{N+2})\gamma_z(x,t)
\end{align*}
\]  

(12)

where \( w_0 \) denotes the transverse displacement; \( \theta_z \), \( \phi_z \) and \( \gamma_z \) denote the generalized displacements signifying the gradients corresponding to \( z \)-direction of HFRC base beam, ACLD and the constraining layer, respectively.

To simplify the mathematical formulation, the generalized displacement variables divided into two vectors are written as

\[
\mathbf{d}_1 = \begin{bmatrix} u_0 & w_0 \end{bmatrix}^T
\]

and

\[
\mathbf{d}_2 = \begin{bmatrix} \theta_x & \theta_z & \phi_z & \phi_z & \gamma_z & \gamma_z \end{bmatrix}^T
\]  

(13)

The normal strains \( (\varepsilon_x^b, \varepsilon_z^b) \) represent the state of strain at any layer of the laminated HFRC smart
beam with respect to the $x$- and $z$-axis, respectively, and $\epsilon_{sx}^{k}$ is the transverse shear strain.

Using equations (11) and (12), the state of strain can be obtained by considering plane strain condition and the relations of linear strain displacement

$$
\{\epsilon_{b}\} = \{\epsilon_{br}\} + [Z_i]\{\epsilon_{hr}\}
$$

The relations of linear strain displacement are obtained by considering plane strain condition and the generalized strains and $\{\epsilon_{hr}\}$, $\{\epsilon_{1}\}$, $\{\epsilon_{2}\}$, $\{\epsilon_{3}\}$, $\{\epsilon_{4}\}$, $\{\epsilon_{5}\}$, and $\{\epsilon_{6}\}$ are the transformation matrices that are written as

$$
\{\epsilon_{b}\} = \{\epsilon_{br}\} + [Z_i]\{\epsilon_{hr}\} \quad (14)
$$

$$
\epsilon_{st} = \epsilon_{st} + [Z_i]\{\epsilon_{sr}\}, \quad k = 1, 2, 3, \ldots, N
$$

$$
\epsilon_{sz} = \epsilon_{st} + [Z_i]\{\epsilon_{sr}\}, \quad k = N + 1
$$

$$
\epsilon_{sz} = \epsilon_{st} + [Z_i]\{\epsilon_{sr}\}, \quad k = N + 2
$$

$$
\{\epsilon_{b}\} = \{\epsilon_{br}\} + [Z_i]\{\epsilon_{hr}\}
$$

$$
\{\epsilon_{br}\} = [Z_i]\{\epsilon_{hr}\}, \quad \{\epsilon_{br}\} = \begin{bmatrix} \frac{\partial \phi_i}{\partial x} & \frac{\partial \phi_i}{\partial y} & \frac{\partial \phi_i}{\partial z} \\ \theta_x & \theta_y & \theta_z \\ \gamma_x & \gamma_y & \gamma_z \end{bmatrix}^T
$$

$$
\epsilon_{st} = \frac{\partial \phi_0}{\partial x},
$$

$$
\epsilon_{sr} = \begin{bmatrix} \theta_x & \phi_x & \gamma_x \\ \theta_z & \phi_z & \gamma_z \\ \theta_y & \phi_y & \gamma_y \end{bmatrix}^T
$$

$$
\{Z_i\} = \begin{bmatrix} z & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ h/2 & (z - h/2) & 0 & 0 & 0 & 0 \\ 0 & h/2 & (z - h/2) & 0 & 0 & 0 \\ 0 & 0 & h/2 & (z - h/2) & 0 & 0 \\ 0 & 0 & 0 & h/2 & (z - h/2) & 0 \end{bmatrix}
$$

$$
\{\epsilon_{b}\} = \left[\begin{array}{c} \epsilon_{1}^{k} \\ \epsilon_{2}^{k} \\ \epsilon_{3}^{k} \\ \epsilon_{4}^{k} \\ \epsilon_{5}^{k} \\ \epsilon_{6}^{k} \end{array}\right]
$$

$$
\{\epsilon_{br}\} = \left[\begin{array}{c} \epsilon_{1}^{k} \\ \epsilon_{2}^{k} \\ \epsilon_{3}^{k} \\ \epsilon_{4}^{k} \\ \epsilon_{5}^{k} \end{array}\right]
$$

$$
\{\epsilon_{hr}\} = \left[\begin{array}{c} \epsilon_{1}^{k} \\ \epsilon_{2}^{k} \\ \epsilon_{3}^{k} \end{array}\right]
$$

where $\{\epsilon_{hr}\}$ is the strain vector, $\{\epsilon_{br}\}$ is the generalized strain, $\{\epsilon_{sr}\}$ is the transverse shear strain.

In which $\{\epsilon_{b}\}$ is the strain vector, $\{\epsilon_{br}\}$, $\{\epsilon_{hr}\}$, $\{\epsilon_{sr}\}$ are the transformation matrices and $[Z_i]$, $[Z_j]$, $[Z_k]$, $[Z_m]$, $[Z_n]$ and $[Z_o]$ are the transformation matrices that are written as

where $\{\epsilon\}$ indicates the 1–3 PZC constant matrix and $E_z$ indicates the applied electric field and these terms can be given by:

$$
\{\epsilon\} = \begin{bmatrix} \epsilon_{31} & \epsilon_{33} \end{bmatrix}^T \quad \text{and} \quad E_z = -V/h_p
$$

where $V$ denotes a voltage difference applied over the thickness of 1–3 PZC layer.

$T_p$ and $T_k$ indicate the total potential and the kinetic energy of the overall beam and obtained as

$$
T_p = \frac{1}{2} \sum_{k=1}^{N+1} \int_{\Omega} \left( \{\epsilon^{k}\}^T \{\sigma^{k}\} + c_{ijkl} \epsilon_{ij}^{k} \epsilon_{ij}^{*k} \right) d\Omega - \frac{1}{2} \int_{\Omega} D_2 E_z d\Omega - \frac{1}{2} \int_{A} \rho w dA
$$

and

$$
T_k = \frac{1}{2} \sum_{k=1}^{N+1} \int_{\Omega} \rho^k \{\ddot{d}_{i}\}^T \{\ddot{d}_{i}\} d\Omega
$$

in which mass density of any $k$th layer is denoted by $\rho^k$; $\rho$ denotes the surface traction applied externally over a surface area $A$ and $\Omega$ indicates the volume. While evaluating the kinetic energy, the rotary inertia was ignored as the beam is considered to be a thin beam. The discretization of the overall beam is carried out by using three noded isoparametric bar element.
Making use of equation (13), the generalized displacement vectors with respect to \( i \)-th (\( i = 1, 2, 3 \)) node of the element are expressed as:

\[
\{d_i\} = \begin{bmatrix} u_{oi} & w_{oi} \end{bmatrix}^T
\]

and

\[
\{d_{ni}\} = \begin{bmatrix} \theta_{xi} & \theta_{zi} & \phi_{xi} & \phi_{zi} & \gamma_{xi} & \gamma_{zi} \end{bmatrix}^T
\]

(21)

Therefore, the generalized displacement vectors at any point within the element are expressed as:

\[
\{d_i\} = [N_i]\{d^e_i\} \quad \text{and} \quad \{d_r\} = [N_i]\{d^r\}
\]

(22)

where

\[
\{d^e_i\} = \begin{bmatrix} \{d^e_{i1}\}^T \{d^e_{i2}\}^T \{d^e_{i3}\}^T \end{bmatrix}^T,
\]

\[
\{d^r\} = \begin{bmatrix} \{d^r_{i1}\}^T \{d^r_{i2}\}^T \{d^r_{i3}\}^T \end{bmatrix}^T.
\]

\[
[N_i] = \begin{bmatrix} N_{i1} & N_{i2} & N_{i3} \end{bmatrix}^T,
\]

\[
[N_i] = \begin{bmatrix} N_{i1} & N_{i2} & N_{i3} \end{bmatrix}^T,
\]

\[
N_{ni} = n_{i1}I_1 \quad \text{and} \quad N_{ni} = n_{i1}I_1
\]

with \( I_1 \) and \( I_3 \) are respective 2 \( \times \) 2 and 6 \( \times \) 6 unit matrices, and \( n_i \) is the shape function of natural coordinates with respect to \( i \)-th node. Employing equations (13), (15), (21) and (22), the generalized strain–displacement matrices that can be written as

\[
B_{tri} = \begin{bmatrix}
\frac{\delta n_{e}}{\delta x} & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{\delta n_{e}}{\delta x} & 0 & 0 \\
0 & 0 & 0 & \frac{\delta n_{e}}{\delta x} & 0 \\
0 & n_{i} & 0 & 0 & 0 \\
0 & 0 & 0 & n_{i} & 0 \\
0 & 0 & 0 & 0 & n_{i}
\end{bmatrix}
\]

and (25)

\[
B_{rsi} = \begin{bmatrix}
n_{i} & 0 & 0 & 0 & 0 \\
0 & n_{i} & 0 & 0 & 0 \\
0 & 0 & n_{i} & 0 & 0 \\
0 & 0 & 0 & n_{i} & 0 \\
0 & 0 & 0 & 0 & n_{i}
\end{bmatrix}
\]

\[
\frac{\delta n_{e}}{\delta x} & 0 & 0 & 0 & 0 \\
0 & \frac{\delta n_{e}}{\delta x} & 0 & 0 & 0 \\
0 & 0 & \frac{\delta n_{e}}{\delta x} & 0 & 0 \\
0 & n_{i} & 0 & 0 & 0 \\
0 & 0 & 0 & n_{i} & 0 \\
0 & 0 & 0 & 0 & n_{i}
\end{bmatrix}
\]

Finally, by making use of equations (22) and (23) into equations (19) and (20), the total potential energy \( (T_p^e) \) and the kinetic energy \( (T_k^e) \) of a typical length of element \( (L_e) \) improved via ACLD treatment and can be written as follows

\[
T_p^e = \frac{1}{2}b \int_{0}^{L_e} \left( \frac{1}{2} \{d^e_i\}^T \{K_n^e\} \{d^e_i\} + \{d^r\}^T \{K_r\} \{d^r\} \\
+ \{d^e_i\}^T \{K_{tr}\} \{d^e_i\} + \{d^r\}^T \{K_{rr}\} \{d^r\} \\
- 2 \{d^e_i\}^T \{F_{ip}\} V - 2 \{d^e_i\}^T \{F_{ip}\} \\
\right) dx
\]

(26a)

and

\[
T_k^e = \frac{1}{2} \int_{0}^{L_e} m \left( \{d^e_i\}^T \{N\}^T \{N\} \{d^e_i\} \right) dx
\]

(26b)

In equations (26a) and (26b), the elemental stiffness matrices \( \{K_n^e\} \), \( \{K_r\} \), \( \{K_{tr}\} \), elemental electrostatic coupling vector \( \{F_{ip}\} \), \( \{F_{ip}\} \), elemental load vector \( \{F^e\} \) and the mass parameter \( (m) \) can be obtained as follows

\[
[K_n^e] = \int_{0}^{L_e} \left( \{B_{nb}\}^T \{D_{nb}\}\{B_{nb}\} + \{B_{nb}\}^T \{D_{nb}\}\{B_{ra}\} \right) dx,
\]

\[
[K_r] = \int_{0}^{L_e} \left( \{B_{rb}\}^T \{D_{rb}\}\{B_{rb}\} + \{B_{rb}\}^T \{D_{rb}\}\{B_{ra}\} \right) dx,
\]

\[
[K_{tr}] = \int_{0}^{L_e} \left( \{B_{rb}\}^T \{D_{rb}\}\{B_{rb}\} + \{B_{rb}\}^T \{D_{rb}\}\{B_{ra}\} \right) dx,
\]

\[
[K_{rr}] = \int_{0}^{L_e} \left( \{B_{rb}\}^T \{D_{rb}\}\{B_{rb}\} + \{B_{rb}\}^T \{D_{rb}\}\{B_{ra}\} \right) dx,
\]

The submatrices appeared in equation (24) can be obtained as

\[
B_{bi} = \begin{bmatrix}
\frac{\delta n_{e}}{\delta x} & 0 \\
0 & \frac{\delta n_{e}}{\delta x}
\end{bmatrix},
\]

\[
B_{si} = \begin{bmatrix}
0 & \frac{\delta n_{e}}{\delta x}
\end{bmatrix}.
\]
\[ \{F_{tp}\} = \int_0^{L_l} [B_i]^T \{D_{tp}\} dx; \]
\[ \{F_{tp}\} = \int_0^{L_l} [B_i]^T \{D_{tp}\} dx; \]
\[ \{F_{tp}\} = \int_0^{L_l} p[N_i]^T [0 \ 1]^T dx \]
\[ \bar{m} = \sum_{k=1}^{N_2} \rho k (h_{k+1} - h_k) \]  
(27a)

The various rigidity matrices are denoted by \([D_{b}], [D_{rb}], [D_{rpb}], [D_{n}], [D_{rn}], [D_{m}]\) and the rigidity vectors for electroelastic coupling \((\{D_{tp}\}, \{D_{rp}\})\) appeared in equation (27a) are expressed as follows

\[ [D_{b}] = \sum_{k=1}^{N_2} \int_{h_k}^{h_{k+1}} [C_b]^T dz \]
\[ [D_{rb}] = \sum_{k=1}^{N} \int_{h_k}^{h_{k+1}} [C_b]^T [Z_1] dz + \int_{h_{k+1}}^{h_{k+2}} [C_b]^{N+1} [Z_2] dz \]
\[ + \int_{h_{k+1}}^{h_{k+2}} [Z_2]^T [C_b]^{N+2} [Z_3] dz \]
\[ [D_{rpb}] = \sum_{k=1}^{N} \int_{h_k}^{h_{k+1}} [Z_1]^T [C_b] [Z_1] dz + \int_{h_{k+1}}^{h_{k+2}} [Z_2]^T [C_b]^{N+1} [Z_2] dz \]
\[ + \int_{h_{k+1}}^{h_{k+2}} [Z_3]^T [C_b]^{N+2} [Z_2] dz \]
\[ \{D_{tp}\} = \int_{h_{k+1}}^{h_{k+2}} -\{\varepsilon\} / h_p dz \quad \text{and} \]
\[ \{D_{rp}\} = \int_{h_{k+1}}^{h_{k+2}} -[Z_3]^T \{\varepsilon\} / h_p dz \]  
(27b)

Applying the principle of virtual work, the open loop equation of motion for the coupled beam can be expressed as:

\[ \{M\} \{\ddot{X}\} + [K_{nl}] \{X\} + [K_{tr}] \{X_r\} = \{F_{tp}\} V + \{F\} \]  
(30)

and

\[ [K_{nl}] \{X\} + [K_{tr}] \{X_r\} = \{F_{tp}\} V \]  
(31)

where the global mass matrix is denoted by \([M]\); the global stiffness matrices are represented by \([K_{nl}], [K_{tr}]\); the global electroelastic coupling vectors are denoted by \({F_{tp}}\) and \({F_{rp}}\); the global nodal generalized displacement vectors are denoted by \({X}\) and \({X_r}\) and the global nodal force vector is presented by \({F}\). After the application of boundary conditions, the global equation of motion in terms of global nodal translational degrees of freedom (DOF) \({X}\) is obtained by reducing the global generalized DOF \({X_r}\) and can be written as follows

\[ \{M\} \{\ddot{X}\} + [K] \{X\} = \left(\{F_{tp}\} - [K_{tr}]^{-1} \{F_{tp}\}\right) V + \{F\} \]  
(32)

where the global stiffness matrix \([K]\) can be formulated as

\[ [K] = [K_{nl}] - [K_{tr}]^{-1} [K_{tr}] \]

When voltage difference is not supplied to the 1–3 PZC, then equation (32) can be used to show the uncontrolled (passive) constrained layer damping of the HFRC base beam.

**Closed-loop model**

A closed-loop model is presented to calculate the required control voltage for active damping proportional to the velocity at the free end of the beam. In the case of active control, the control voltage \((V)\) is applied to the translational velocity at a point \((L_s, 0)\) in terms of the derivatives of the global nodal translational degrees of freedom can be written as

\[ V = -k_d \dot{w}(L_s, 0) = -k_d [U]^T \{\ddot{X}\} \]  
(33)
where the required control gain is represented by \(k_d\) and row vector \([U]\) is denoting the sensor location.

Making use of equation (33) into equation (32), the closed-loop characteristics of smart HFRC beam system can be obtained using equation of motion

\[
[M]\{\ddot{X}\} + [C_d]\{\dot{X}\} + [K]\{X\} = \{F\}
\]

(34)

where \([C_d]\) denotes the matrix of active damping and can expressed as

\[
[C_d] = k_d \left( [F_{tp}] - [K_{tr}] [K_{tr}]^{-1} [F_{tp}] \right) [U]
\]

(35)

Results and discussion

In this section, the numerical outcomes are presented for the effective elastic properties of HFRC determined using the two- and three-phase analytical MOM model as presented in Effective elastic properties of HFRC section. For the active control of a laminated smart beam, we obtained the results using the FE model derived in FE modelling of a smart beam section.

**Effective elastic properties of HFRC**

To determine the effective elastic properties of the HFRC, epoxy and carbon fibre are used as the matrix and fibre material, respectively. We mixed an adequate amount of CNT with epoxy to improve the mechanical and damping property of the composite and its structures. The mechanical properties of the epoxy, carbon fibre and CNT nanofiller are enlisted in Table 1.

For the development of lightweight and high-performance composite, the maximum \(v_{CNT}\) in the epoxy matrix can vary practically up to 27% when used with different conventional fibres via one of the novel fabrication methods such as shear pressing.\(^{49,50,64}\) The \(v_{CF}\) in the composite can vary from 0.2 to 0.8. Unless otherwise mentioned, CNT (5, 5) is used to evaluate the mechanical properties of the HFRC using the MOM model, and the estimated results are shown in Figures 4 to 9. In the current study, we estimated the results for 0 and 10% of \(v_{CNT}\) in the composite. It should be noted that to predict the mechanical properties of the base composite, the two-phase MOM model is used for which \(v_{CNT} = 0\) and for the HFRC, three-phase MOM model is used when CNT (\(v_{CNT} = 10\%\)) is added to the composite.

**Table 1. Properties of the nanofiller, fibre and matrix**

<table>
<thead>
<tr>
<th>Material</th>
<th>Ref.</th>
<th>(C_{11}) (GPa)</th>
<th>(C_{12}) (GPa)</th>
<th>(C_{13}) (GPa)</th>
<th>(C_{23}) (GPa)</th>
<th>(C_{33}) (GPa)</th>
<th>(C_{44}) (GPa)</th>
<th>(\rho) (kg/m(^3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>CNT (5,5)</td>
<td>31</td>
<td>668</td>
<td>404</td>
<td>184</td>
<td>184</td>
<td>2153</td>
<td>791</td>
<td>1400</td>
</tr>
<tr>
<td>CNT (10,10)</td>
<td>28</td>
<td>288</td>
<td>254</td>
<td>87.8</td>
<td>87.8</td>
<td>1088</td>
<td>442</td>
<td>1400</td>
</tr>
<tr>
<td>CNT (10,0)</td>
<td>59</td>
<td>570.9</td>
<td>172.4</td>
<td>240</td>
<td>240</td>
<td>1513.1</td>
<td>1120</td>
<td>1400</td>
</tr>
<tr>
<td>CNT (14,0)</td>
<td>59</td>
<td>557.5</td>
<td>137.5</td>
<td>187.7</td>
<td>187.7</td>
<td>1082.8</td>
<td>779.2</td>
<td>1400</td>
</tr>
<tr>
<td>Carbon fibre</td>
<td>59</td>
<td>236.4</td>
<td>10.6</td>
<td>10.6</td>
<td>10.7</td>
<td>24.8</td>
<td>7</td>
<td>1700</td>
</tr>
<tr>
<td>Epoxy</td>
<td>28</td>
<td>5.3</td>
<td>3.1</td>
<td>3.1</td>
<td>3.1</td>
<td>5.3</td>
<td>0.64</td>
<td>1250</td>
</tr>
</tbody>
</table>

CNT: carbon nanotube.
Figure 4 shows the comparison of the effective longitudinal elastic constant \( C_{11} \) with respect to \( v_{CF} \) of HFRC and base composite. We obtained almost a linear curve as \( C_{11} \) varies with \( v_{CF} \), this is mainly because the iso-strain condition was considered with the axis of symmetry for the prediction of effective longitudinal elastic constant \( C_{11} \), a similar trend was obtained by Kundalwal and Ray.\(^59\) As the value of \( v_{CNT} \) increases from 0% to 10%, a significant improvement in the elastic constant \( C_{11} \) of the HFRC can be observed due to the incorporation of CNT nanofiller in the pure matrix as shown in Figure 4.

The comparison of the effective transverse elastic constant \( C_{33} \) with \( v_{CF} \) of the HFRC and base composite are illustrated in Figure 5. For the variation of CNT from 0% to 10%, at the lower value of \( v_{CF} \) less improvement in \( C_{33} \) is observed. Whereas comparatively high enhancement may be seen in \( C_{33} \), as the value of \( v_{CF} \) increases. Similar results were observed for \( C_{22} \), however, for the sake of brevity the results are not present here. Since, the material is considered as transversely isotropic along the longitudinal direction; therefore, due to the constructional feature of both HFRC and base composite, the two results \( (C_{33} \ and \ C_{22}) \) are similar. From Figures 4 and 5, it may be observed that for the given \( v_{CF} \), the values of \( C_{11} \) are much higher as compared to the values of \( C_{33} \) in magnitude. This is because these transverse elastic constants \( C_{33} \) are matrix dependent. Since the CNTs and carbon fibres are aligned in the longitudinal axis, the longitudinal stiffness of the HFRC lamina is improved. Similarly, the effective elastic constant \( C_{12} \) and \( C_{13} \) are computed using similar predictions as shown in Figures 6 and 7.

Figure 8 illustrates the comparison of the effective elastic constant \( C_{23} \) with respect to \( v_{CF} \). For the lower value of \( v_{CF} \), there is a slight increment in the results obtained for the HFRC as compared to the base composite. Whereas for the higher value of \( v_{CF} \), a significant enhancement may be observed. The trend of result for \( C_{23} \) and \( C_{13} \) are found to be almost similar, although for predicting the \( C_{23} \) the normal strain \( (e_{23}) \) and normal stress \( (\sigma_{22}) \) are under the extension–extension coupling, as the loading condition was considered along the longitudinal direction. Figure 9 depicts the comparison of the effective shear elastic constant \( C_{44} \) with \( v_{CF} \) of the HFRC and base composite. It may be noted that the \( C_{44} \) follows the same trend as \( C_{33} \) and \( C_{23} \), mainly, due to the dependency of \( C_{44} \) on the values of effective elastic constants \( C_{33} \) and \( C_{23} \). It may be noted that the earlier researcher\(^{65}\) reported similar outcomes for predicting effective elastic constant \( C_{44} \). In their study, they found that the longitudinal shear modulus \( C_{44} \) of the laminated composite computed with the analytical model can be changed considerably compared to experimental values.

From Figures 4 to 9, it may be concluded that the effective axial, transverse and shear elastic properties of the HFRC are improved by incorporation of
CNTs into the epoxy matrix, due to the high elastic properties of CNTs and the non-bonded interaction formed between CNTs and the epoxy phase. Therefore, for investigating the performance of a smart beam, these properties are utilized which were estimated by using the MOM model.

**Active damping of HFRC smart cantilever beam**

To study the performance of the laminated HFRC as a damping material for a smart structure, numerical outcomes are estimated employing the FE model developed in earlier FE modelling of a smart beam section. The presented FE model is developed using in-house code based on FSDT. The advantage of this theory is that it uses shear lag correction factor to overcome the problem of constant shear-stress. Note that one cannot model irregular shape of structure using such in-house FE formulation and another type of FE approach or commercial FE software needs to be employed to model irregular structure.

![Figure 9. Comparison of $C_{44}$ with respect to $v_{CF}$ for HFRC and base composite.](image)

For the active damping, the frequency responses of the smart cantilever composite beam are calculated. For this, equation (27a) is formulated with the time-harmonic point load of 2 N subjected at the tip of the free end to investigate amplitude and frequency response analysis. Further analysis has been carried out by considering the carbon fibre volume fraction as 40% in base composite and HFRC. For 1–3 PZC, the fibre volume fraction is taken as 60%. The material properties of the PZC, base composite and HFRC are tabulated in Tables 2 and 3.

The length of the transversely isotropic HFRC substrate beam is taken as 0.5 m, whereas the thickness of each layer of the HFRC substrate, piezoelectric layer and viscous layer are considered as 0.005 m, 0.001 m and 0.0002 m, respectively. In FE modelling, we adopted shear correction factor of 5/6 in FSDT which is commonly used in for thin plates.

The length and thickness of each layer of transversely isotropic HFRC substrate beam are taken as 0.5 m and 0.005 m, respectively. The viscous layer and piezoelectric layer are having a thickness of 0.0002 m and 0.001 m, respectively, and the length of ACLD patch is covering 3/5th of the beam. The properties of the viscoelastic constraining layer such as density, Poisson’s ratio and complex shear modulus are considered as 1140 kg/m$^3$, 0.3 and $20(1+i)$ MN m$^{-2}$, respectively.

To verify the validation of the FE model derived in FE modelling of a smart beam section for active damping of laminated HFRC substrate beam integrated with ACLD treatment constraining layer of 1–3 PZC, we compare the frequency response with existing models for the identical beam adopting the similar boundary conditions. These results are tabulated in Table 4 for symmetric cross-ply ($0^\circ/90^\circ/0^\circ$) and anti-symmetric angle-ply ($-45^\circ/45^\circ/-45^\circ/45^\circ$) substrate. Table 4 shows a good agreement between the present FE model and the existing literature. The

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**Table 2. Properties of the piezoelectric material.**

<table>
<thead>
<tr>
<th>Material</th>
<th>$C_{11}$ (GPa)</th>
<th>$C_{13}$ (GPa)</th>
<th>$C_{33}$ (GPa)</th>
<th>$C_{44}$ (GPa)</th>
<th>$\varepsilon_{31}$ (C m$^{-2}$)</th>
<th>$\varepsilon_{33}$ (C m$^{-2}$)</th>
<th>$\rho$ (kg/m$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1–3 PZC</td>
<td>9.293</td>
<td>6.182</td>
<td>35.444</td>
<td>1.536</td>
<td>$-0.19$</td>
<td>18.41</td>
<td>5090</td>
</tr>
<tr>
<td>PZT-5H</td>
<td>151</td>
<td>96</td>
<td>124</td>
<td>23</td>
<td>$-5.1$</td>
<td>27</td>
<td>7750</td>
</tr>
</tbody>
</table>

PZT: lead zirconate titanate.

**Table 3. Effective elastic properties of HFRC lamina.**

<table>
<thead>
<tr>
<th>Eff. elastic constant</th>
<th>Two-phase ($v_{CNT} = 0%$)</th>
<th>Three-phase ($v_{CNT} = 5%$)</th>
<th>Three-phase ($v_{CNT} = 10%$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$v_{CF} = 40%$</td>
<td>$v_{CF} = 60%$</td>
<td>$v_{CF} = 40%$</td>
</tr>
<tr>
<td>$C_{11}$ (GPa)</td>
<td>96.6451</td>
<td>142.5567</td>
<td>125.5864</td>
</tr>
<tr>
<td>$C_{12}$ (GPa)</td>
<td>4.1219</td>
<td>5.0647</td>
<td>4.2920</td>
</tr>
<tr>
<td>$C_{13}$ (GPa)</td>
<td>4.1219</td>
<td>5.0647</td>
<td>4.2719</td>
</tr>
<tr>
<td>$C_{23}$ (GPa)</td>
<td>4.3856</td>
<td>5.521</td>
<td>4.7174</td>
</tr>
<tr>
<td>$C_{33}$ (GPa)</td>
<td>7.7068</td>
<td>9.9779</td>
<td>8.3094</td>
</tr>
<tr>
<td>$C_{44}$ (GPa)</td>
<td>1.0054</td>
<td>1.407</td>
<td>1.091</td>
</tr>
</tbody>
</table>
The convergence study of laminated HFRC beam integrated with the ACLD treatment is presented in Table 5. Considering the convergence study, the frequency response analysis of laminated beam is done by meshing the beam into 20 elements. Figure 10 illustrates the uncontrolled variation of amplitude $w_{L,0}$ with the frequency response of $0^\circ/90^\circ/0^\circ$ HFRC substrate beam for different percentage of CNT $(5, 5)$ ($v_{CNT} = 0$ and $10\%$). The figure illustrates that the laminated HFRC attenuates the amplitude of deflection of cantilever beam at free end significantly for the small value of $v_{CNT}$ and thus, a slight enhancement in the damping characteristic of the system is observed for the passive damping (uncontrolled or gain = 0). For designing a smart structure, the fundamental frequency (i.e., the first mode of natural frequency) is an important parameter to be considered. The amplitude of fundamental frequency decreases from $3.49 \times 10^{-4}$ m to $3.46 \times 10^{-4}$ m (see Table 6) due to the incorporation of CNTs ($v_{CNT} = 10\%$). It may be observed from passive damping that slight attenuation in the amplitudes of deflection is achieved with laminated HFRC beam. It will be very interesting to see the active performance of laminated HFRC beam and hence, the further analysis of the laminated HFRC beam for active damping (gain $\neq 0$) and control voltage are shown in Figures 11 and 12. The frequency v/s amplitude of deflection $w_{L,0}$ of $0^\circ/90^\circ/0^\circ$ HFRC and base composite substrate beam is shown in Figure 11. This figure depicts both uncontrolled (passive damping or gain = 0) and controlled (active damping or gain $\neq 0$) gain. It may be seen that for the given gain, the HFRC substrate attenuates the amplitude of deflection significantly as compared to the base composite. Table 6 summarizes the values of the maximum amplitude corresponding to the fundamental natural frequency for the first mode. From Table 6 and Figure 11, it may be observed that the reduction in amplitude of the HFRC substrate beam is $\sim 10\%$ and $\sim 12\%$ corresponding to the value of gain ($k_{s}$) = 2000 and 3000. Due to the incorporation of CNTs, both stiffness and density of the HFRC substrate enhanced. In particular, the higher stiffness of the HFRC substrate beam reduces the amplitudes of deflection. Hence, higher damping is achieved in case of deflection is achieved with laminated HFRC beam.

### Table 4. Frequency response ACLD integrated beam.

<table>
<thead>
<tr>
<th>Beams Source</th>
<th>1st mode</th>
<th>2nd mode</th>
<th>3rd mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0^\circ/90^\circ/0^\circ$</td>
<td>Present FEM</td>
<td>34</td>
<td>204</td>
</tr>
<tr>
<td>Ray and Pradhan$^{55}$</td>
<td>33</td>
<td>207</td>
<td>568</td>
</tr>
<tr>
<td>Ray and Malik$^{67}$</td>
<td>35</td>
<td>210</td>
<td>572</td>
</tr>
<tr>
<td>$-45^\circ/45^\circ/ -45^\circ/45^\circ$</td>
<td>Present FEM</td>
<td>20.3</td>
<td>114.8</td>
</tr>
<tr>
<td>Ray and Pradhan$^{55}$</td>
<td>18.3</td>
<td>113.5</td>
<td>314.6</td>
</tr>
<tr>
<td>Ray and Malik$^{67}$</td>
<td>19.2</td>
<td>115.6</td>
<td>316.7</td>
</tr>
<tr>
<td>Al substrate beam</td>
<td>Present FEM</td>
<td>84</td>
<td>266</td>
</tr>
<tr>
<td>Experimental$^{48}$</td>
<td>92</td>
<td>258.5</td>
<td>502</td>
</tr>
</tbody>
</table>

FEM: finite element model.

### Table 5. Convergence study of HFRC beam integrated with ACLD treatment.

<table>
<thead>
<tr>
<th>Beams Elements</th>
<th>Frequency (Hz)</th>
<th>1st mode</th>
<th>2nd mode</th>
<th>3rd mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0^\circ/90^\circ/0^\circ$</td>
<td>5</td>
<td>32</td>
<td>169</td>
<td>454</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>33</td>
<td>171</td>
<td>471</td>
</tr>
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<td></td>
<td>15</td>
<td>33</td>
<td>172</td>
<td>475</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>33</td>
<td>172</td>
<td>475</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>33</td>
<td>172</td>
<td>475</td>
</tr>
<tr>
<td>$0^\circ/90^\circ/0^\circ/90^\circ$</td>
<td>5</td>
<td>5</td>
<td>120</td>
<td>324</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>24</td>
<td>122</td>
<td>337</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>24</td>
<td>123</td>
<td>339</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>24</td>
<td>123</td>
<td>340</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>24</td>
<td>123</td>
<td>340</td>
</tr>
<tr>
<td>$-45^\circ/45^\circ/ -45^\circ/45^\circ$</td>
<td>5</td>
<td>18</td>
<td>97</td>
<td>263</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>19</td>
<td>99</td>
<td>273</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>19</td>
<td>99</td>
<td>275</td>
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<tr>
<td></td>
<td>20</td>
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<td>99</td>
<td>275</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>19</td>
<td>99</td>
<td>275</td>
</tr>
</tbody>
</table>

Figure 10. Variation of uncontrolled amplitude of deflection with respect to frequency response of $0^\circ/90^\circ/0^\circ$ HFRC substrate beam.
of HFRC substrate beam as compared to the base composite beam. This reveals that the addition of CNTs enhances the damping characteristic of the HFRC substrate beam for controlled or active damping.

Table 6. Amplitudes and fundamental natural frequencies of HFRC substrate smart beam corresponding to ply type.

<table>
<thead>
<tr>
<th>Ply</th>
<th>Gain</th>
<th>(v_{\text{CNT}} = 0)</th>
<th>Amplitude (m)</th>
<th>Frequency (1st mode) (Hz)</th>
<th>Amplitude (m)</th>
<th>Frequency (1st mode) (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0^\circ/90^\circ/0^\circ)</td>
<td>Uncontrolled</td>
<td>3.49 \times 10^{-4}</td>
<td>26</td>
<td>3.46 \times 10^{-4}</td>
<td>33</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2000</td>
<td>2.11 \times 10^{-4}</td>
<td>26</td>
<td>1.92 \times 10^{-4}</td>
<td>33</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3000</td>
<td>1.77 \times 10^{-4}</td>
<td>26</td>
<td>1.57 \times 10^{-4}</td>
<td>33</td>
<td></td>
</tr>
<tr>
<td>(0^\circ/90^\circ/0^\circ/90^\circ)</td>
<td>Uncontrolled</td>
<td>3.55 \times 10^{-4}</td>
<td>19</td>
<td>3.51 \times 10^{-4}</td>
<td>24</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2000</td>
<td>2.28 \times 10^{-4}</td>
<td>19</td>
<td>2.09 \times 10^{-4}</td>
<td>24</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3000</td>
<td>1.95 \times 10^{-4}</td>
<td>19</td>
<td>1.76 \times 10^{-4}</td>
<td>24</td>
<td></td>
</tr>
<tr>
<td>(-45^\circ/45^\circ/ -45^\circ/45^\circ)</td>
<td>Uncontrolled</td>
<td>3.52 \times 10^{-4}</td>
<td>16</td>
<td>3.48 \times 10^{-4}</td>
<td>19</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2000</td>
<td>2.59 \times 10^{-4}</td>
<td>16</td>
<td>2.40 \times 10^{-4}</td>
<td>19</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3000</td>
<td>2.30 \times 10^{-4}</td>
<td>16</td>
<td>2.11 \times 10^{-4}</td>
<td>19</td>
<td></td>
</tr>
</tbody>
</table>

Figure 11. Variation of amplitude of deflection with frequency response of \(0^\circ/90^\circ/0^\circ\) HFRC substrate beam.

Figure 12. Variation of control voltage of \(0^\circ/90^\circ/0^\circ\) HFRC substrate beam with frequency.

Figure 13. Effect of piezoelectric constant \(e_{33}\) on the amplitude of deflection \(0^\circ/90^\circ/0^\circ\) HFRC substrate beam with value of \(k_d = 2000\).

Figure 12 illustrates the voltage required to control the different modes of the amplitude of the deflection of the cantilever HFRC substrate beam at the free end. It may be seen from Figure 12 that for the HFRC substrate beam, the maximum control voltage for the value of gain \(k_d = 2000\) is 71.33 V, which is \(\sim 15\%\) less as compared to the voltage required for base composite. The difference in control voltage required for the HFRC and base composite substrate beam is considerably high. In practice, such a less value of control voltage can be easily arranged.

Figure 13 illustrates the damping performance of smart structure for the value of gain = 2000, with and without considering the piezoelectric constant \(e_{31}\) and \(e_{33}\). From Figure 13, it may be seen that when the value of the piezoelectric constant \(e_{31} \neq 0\) and \(e_{33} = 0\), much higher amplitudes of deflections are observed when compared to the case of \(e_{31} = 0\) and \(e_{33} \neq 0\). From this, it may be concluded that the constraining layer of ACLD is mainly responsible for the attenuation of vertical shear deformation. This also means that the out-of-plane actuation has a much
higher contribution in vibration attenuation than in-plane actuation of the laminated smart beam. In both cases of $e_{33} = 0$ and $e_{33} \neq 0$, the smart beam with the HFRC substrate due to incorporation of CNTs ($v_{CNT} = 10\%$) shows better damping behavior.

Figure 14 illustrates the performance of 1–3 PZC over PZT-5H on the uncontrolled amplitude of deflection of the 0°/90°/0° HFRC smart beam.

To investigate the performance of anti-symmetric cross-ply (0°/90°/0°/90°) HFRC substrate beam, the frequency response for both passive and active control with respect to the amplitude of deflection of cantilever smart beam at free end w(L, 0) are demonstrated in Figure 15. It is noticed that the first mode shows much higher peaks compared to the other modes of natural frequency which is highly controlled for the 0°/90°/0°/90° HFRC substrate beam over the passive damping (gain = 0). The HFRC substrate beam shows the improved damping characteristic and the modes of natural frequency shifted slightly towards the higher side of the frequency as compared to the base composite substrate beam, this difference increases for the higher modes of natural frequency.

Figure 16. Variation of amplitude of deflection with respect to frequency response of $-45°/45°/–45°/45°$ HFRC substrate beam.

To determine the performance of anti-symmetric angle-ply ($–45°/45°/–45°/45°$) HFRC substrate beam, the frequency response with respect to the amplitude of deflection of cantilever smart beam at the free end w(L, 0) are shown in Figure 16. It may be observed that for the given gain the HFRC substrate beam attenuates the amplitude of deflection significantly as compared to base composite substrate beam. For the gain of 2000, the second mode of natural frequency showing higher peaks; however, for the effective control of higher mode, one can change the value of gain. It is clear from Figure 16 that by changing the value of gain from 2000 to 3000 the higher modes are controlled significantly.

Figures 17 and 18 illustrate the voltage required to control the different modes of the amplitude of deflection of cantilever 0°/90°/0°/90° and $-45°/45°/–45°/45°$ HFRC substrate beam at the free end. The estimated value for the maximum control voltage required for 0°/90°/0°/90° base composite beam is 87.71 V and 69.95 V for the HFRC beam, the difference between the two values is $\sim 20\%$, which shows
substantial improvement. Similarly, from Figure 18, the estimated value for the maximum control voltage required for $-45^\circ/45^\circ$ base composite beam is 182.2 V and 128.4 V for HFRC beam, the difference between the two values is $\sim 30\%$. It is noticed that the control voltage required for $-45^\circ/45^\circ$ ply is significantly high, this due to the higher amplitude of deflection observed in Figure 16.

Parametric analysis of $0^\circ/90^\circ/0^\circ$ HFRC substrate beam

Figure 19 illustrates the effect of change in the effective length of the constraining layer on the frequency response with respect to the amplitude of deflection of symmetric cross-ply HFRC beam ($k_d = 2000$). The figure shows that the laminated HFRC beam attenuates the amplitude of deflection significantly as the effective length of the PZC patch ($L_a$) increased from 60% to 80% of the beam length (L). When the value of effective length changes from $L_a = 0.6L$ to $L_a = 0.8L$, the maximum value of the amplitude of the first mode decreases from $2.1 \times 10^{-4} m$ to $1.24 \times 10^{-4} m$, respectively for base composite. Whereas for HFRC, the amplitude decreases from $1.9 \times 10^{-4} m$ to $1.13 \times 10^{-4} m$. This indicates that for $L_a = 0.8L$, the damping characteristics of smart beam enhance approximately around 40% for both base composite and HFRC beam. Whereas for the second mode when the value of effective length changes from $L_a = 0.6L$ to $L_a = 0.8L$, the improvement in the damping characteristic is $\sim 70\%$ and $\sim 60\%$ for base composite and HFRC beam, respectively. Although the percentage reduction for the second mode in the case of HFRC beam is slightly less compared to the base composite beam, the
laminated HFRC beam overall shows better damping characteristics. From this, we can conclude that the damping characteristic of the system can be improved by increasing the effective length of piezoelectric composite material.

The effect of the thickness of PZC patch on frequency response for the amplitude of the deflection of the smart beam is demonstrated in Figure 20. Here, it can be noted that the HFRC substrate beam shows improved damping characteristics as compared to the base composite beam. By increasing the thickness of PZC layer the amplitudes of first mode attenuates approximately by \( \approx 5\% \) for both HFRC and base composite beam; hence, the damping characteristics improves significantly. Although for the second mode, it may be seen from Figure 20 that by increasing the effective thickness of PZC layer results in the higher amplitudes, also a significant change in the natural frequency of the beam. For base composite substrate beam, the frequency shifted from 139 to 130 Hz and from 171 to 158 Hz for the HFRC substrate beam when thickness changed from 0.001 to 0.002 m, respectively. Figure 21 demonstrates the effect of force on the frequency response for the amplitude of the deflection of the cantilever HFRC smart beam at the free end. Many times, these structures are subjected to variable loading conditions, thus it is important to understand the effect of force on the frequency response for the amplitude of deflection while designing the smart structure as a sensor and actuator. Figure 21 illustrates higher amplitude (twice as compared to the amplitude when force \( F = 2 \) N), as the magnitude of force increases from 2 to 4 N and shows the substantial reduction of the amplitude of deflection with consideration of CNTs (\( \nu_{\text{CNT}} = 10\% \)). In actual practice, this variation might not be twice, but it can be concluded that increase in the magnitude of force will result in higher amplitudes.

**Quantitative relative performance of HFRC substrate beam**

In this section, we present the quantitative analysis of HFRC substrate beam for the various \( \nu_{\text{CF}} \) and aspect ratio (\( L/h \)) of the beam. The quantitative analysis is important for selecting the optimum range of fibre in the composite material and aspect ratio of the structure to achieve the maximum efficiency of the overall structure. Figure 22 demonstrates the effective elastic-coefficient enhancement factor of longitudinal elastic constant (\( C_{11} \)) and transverse elastic constant (\( C_{33} \)) with respect to the volume fraction of the carbon fibre (\( \nu_{\text{CF}} \)). The effective elastic-properties enhancement factor (EEEF) shows the percentage enhancement of the effective elastic constants of the laminated HFRC...
where $C_{\text{CNT}}$ denotes the effective elastic-coefficient of the HFRC substrate and $C_0$ represent the effective elastic-coefficient of the base composite.

It may be observed from Figure 22(a) that the EEEF of $C_{11}$ decreases as the $v_{CF}$ increases from 0.2 to 0.8 for the incorporation of 5% and 10% $v_{CNT}$ in the NHCFEC substrate. For $v_{CNT} = 5\%$, the EEEF of $C_{11}$ decreases from $\sim 57\%$ to $\sim 15\%$ and for $v_{CNT} = 10\%$, the EEEF of $C_{11}$ decreases from $\sim 114\%$ to $\sim 31\%$ as the $v_{CF}$ increases from 0.2 to 0.8, respectively. For the longitudinal elastic constant ($C_{11}$), a linear curve is observed (see Figure 4) as the carbon fibres and CNTs are oriented in the same direction. As a result, the EEEF of $C_{11}$ is maximum when $v_{CF}$ is 0.2 and decreases as $v_{CF}$ increases. Thus, to achieve the higher EEEF of $C_{11}$, the $v_{CF}$ should be kept within the range of 0.2 to 0.5. Whereas Figure 22(b) illustrates that EEEF of transverse elastic constant ($C_{33}$) increases significantly, as the $v_{CF}$ increases from 0.2 to 0.8 by incorporating 5% and 10% $v_{CNT}$ in the HFRC substrate beam. The EEEF of $C_{33}$ increases from $\sim 6\%$ to $\sim 16\%$ and $\sim 13\%$ to $\sim 37\%$ for $v_{CNT} = 5\%$ and 10\%, respectively, correspondingly to the value of $v_{CF} = 0.2$ to 0.8. This is because the $C_{33}$ coefficient is matrix dependent and as the $v_{CF}$ increases the fibre becomes more dominant. Thus, the EEEF of $C_{33}$ increases with $v_{CF}$. Therefore, to achieve the higher EEEF of $C_{33}$, optimum range of $v_{CF}$ should be kept within 0.5 to 0.8.

To demonstrate the enhancement in the damping characteristic of the HFRC laminated substrate beam, a damping characteristic enhancement factor (DCEF) is defined. This enhancement factor DCEF shows the reduction in amplitudes of deflection in terms of percentage of the HFRC substrate beam as compared to the base composite and given by

\[
\text{EEEF} = \frac{C_{\text{CNT}} - C_0}{C_0} \times 100\% \quad (36)
\]
compared to the base composite beam and hence, DCEF can be given by

$$DCEF = \frac{A_0 - A_{CNT}}{A_0} \times 100\%$$  \hspace{1cm} (37)$$

where $A_0$ is the amplitude of deflection of base composite beam and $A_{CNT}$ is the amplitude of deflection of the HFRC beam.

Figure 23 illustrates the DCEF of various ply for the first mode of amplitudes of deflection at $v_{CNT} = 5$ and 10%. The results were obtained at the value of gain $(k_d) = 2000$ and it may be seen from the Figure that a similar trend like Figure 22 (a) was observed as the $v_{CF}$ increases the DCEF decreases. This is mainly because the carbon fibre and CNTs are aligned in the first direction and thus $C_{11}$ of the composite has a major impact on the attenuation of vibrational amplitudes. It may also be observed from Figure 23 that the significant enhancement in the damping characteristics achieved by incorporating 5% and 10% $v_{CNT}$ in the base composite. The maximum value of DCEF in case of symmetric and anti-symmetric cross-ply for $v_{CNT} = 5\%$ and 10\% was around ~8\% and 13\%, respectively at $v_{CF} = 0.2$ as shown in Figure 23(a) and (b). Similarly, for anti-symmetric angle-ply (see Figure 23(c)), the maximum value of the DCEF is also observed at $v_{CF} = 0.2$ which is approx. ~6\% and ~10\% for $v_{CNT} = 5\%$ and 10\%, respectively. This value of DCEF decreases as $v_{CF}$ increases from 0.2 to 0.8 as shown in Figure 23. From these results, we can conclude that the HFRC substrate beam is very much effective at lower $v_{CF}$. Also, it is noticed that the attenuation in amplitudes of vibration obtained at higher $v_{CF}$ of the base composite could be achievable with the HFRC at a much lower $v_{CF}$ by considering a certain volume fraction of CNTs.

The structural parameter such as mass, aspect ratio (length, width and thickness) and stiffness affect the natural frequency and vibrational amplitudes of the system. The effect of length to thickness ratio (L/h) on DCEF of the HFRC laminated beam for the first

**Figure 24.** Variation of DCEF with respect to the aspect ratio of the substrate beam for various ply type (a) 0°/90°/0°, (b) 0°/90°/90°/0° and (c) -45°/45°/-45°/45° at a gain $(k_d)$ of 2000 and $v_{CNT} = 5$ and 10%.
mode of amplitudes of vibration is demonstrated in Figure 24. It may be seen from figure that the DCEF enhanced as the aspect ratio decreases and it is maximum at the $L/h = 20$. It was observed, as the aspect ratio of the substrate decreases, the amplitudes of vibration attenuate and the frequency of the overall system improved. This is due to the inherent property of CNTs. CNTs have excellent strength to weight ratio, thus the stiffness of the HFRC substrate enhanced significantly; therefore, the damping performance of the laminated HFRC improved as compared to the base composite.

The results demonstrated in Figures 4 to 24 clearly show the significant enhancement in the damping characteristics of the novel smart HFRC laminated cantilever beam with consideration of CNTs. The quantitative relative performance of HFRC and base composite can be presented by defining two factors: (i) EEEF for the change in effective elastic properties and (ii) DCEF for the change in damping characteristics, which are shown as a bar chart in Figures 22 to 24. Based on these results, one can determine the required response of smart structures by varying different parameters such as length and thickness of piezoelectric patch, applied force, angle of composite ply, aspect ratio of the composite beam and the volume fraction of fibre and nanofillers.

**Conclusion**

An investigation on the damping characteristics of the HFRC laminated smart cantilever beam is performed. The effect of CNTs on the damping characteristic of the laminated HFRC smart beam is analyzed. First, using the approach of mechanics of material, we developed a two-three phase MOM model to evaluate the effective properties of base composite and HFRC by incorporating CNTs in the epoxy phase. Second, the FE model is developed to study the active damping of symmetric and anti-symmetric cross-ply and anti-symmetric angle-ply HFRC smart composite beam attached with ACLD patch treatment. From the present investigation, following main conclusions are carried out:

1. It is observed that due to consideration of the $v_{\text{CNT}}$ the effective properties of the HFRC lamina enhanced notably.
2. The damping characteristics of the novel HFRC laminated beam considering the $v_{\text{CNT}} = 10\%$ for both symmetric and anti-symmetric cross-ply are improved when compared to the base composite with $v_{\text{CNT}} = 0\%$. This is mainly attributed to the high stiffness of HFRC due to the incorporation of CNTs. This implies that the HFRC is a suitable candidate for developing a high-performance composite structure.
3. It may be concluded from the control voltage results that considerably low voltage is required to control the amplitude of HFRC smart beam as compared to the base composite beam.
4. Among the various ply, the performance of $(0^\circ/90^\circ/0^\circ)$ symmetric cross-ply is better considering both the passive and active damping.
5. The contribution of vertical actuation of ACLD treatment constraining the layer of 1–3 PZC material in the vibration damping is significantly higher as compared to in-plane actuation.
6. The performance of tailor made 1–3 PZC is better as compared to PZT-5H, making 1–3 PZC a suitable candidate for the development of the efficient smart structure.
7. It may be concluded from the parametric analysis that the effective length of 1–3 PZC layer is a more sensitive parameter as compared to the effective thickness of 1–3 PZC layer. Because comparatively high enhancement in the damping characteristic of HFRC is observed for the change in the effective length of 1–3 PZC layer.
8. In the quantitative relative performance, we defined two factors: (i) EEEF and (ii) DCEF to analyse the enhanced performance of the laminated HFRC smart beam at various parameters. On the basis of these factors, it is observed that the same performance can be achieved with the HFRC substrate beam with adequate amount of CNTs at much lower range of $v_{\text{CF}}$ which was obtained at the higher range of $v_{\text{CF}}$ in case of base composite substrate beam.
9. From the quantitative analysis, optimum performance of the laminated HFRC smart beam is observed at a range of 0.2 to 0.5 $v_{\text{CF}}$.

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**Supplemental material**

Supplemental material for this article is available online.

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