Analysis of solidification in a finite PCM storage with internal fins by employing heat balance integral method

Rohit Kothari1 | Sreetam Das2 | Santosh K. Sahu1 | Shailesh I. Kundalwal1

1 Discipline of Mechanical Engineering, Indian Institute of Technology Indore, Indore, India
2 Department of Mechanical Engineering, College of Engineering and Technology, Bhubaneswar, Bhubaneswar, India

Correspondence
Rohit Kothari, Discipline of Mechanical Engineering, Indian Institute of Technology Indore, Khandwa Road, Simrol, Indore 453552, India.
Email: rohitkothari2892@gmail.com

Funding information
Department of Science and Technology, Government of India, Grant/Award Number: DST/TMD/MES/2k17/65; DST-INSPIRE Fellowship program, Government of India, Grant/Award Number: IF170534

Summary
Here, a simplified analytical model has been proposed to predict solid fraction, solid–liquid interface, solidification time, and temperature distribution during solidification of phase change material (PCM) in a two-dimensional latent heat thermal energy storage system (LHTES) with horizontal internal plate fins. Host of boundary conditions such as imposed constant heat flux, end-wall temperature, and convective air environment on the vertical walls are considered for the analysis. Heat balance integral method was used to obtain the solution. Present model yields closed-form solution for temperature variation and solid fraction as a function of various modeling parameters. Also, solidification time of PCM, which is useful in optimum design of PCM-based thermal energy storages, has been evaluated during the analysis. The solidification time was found to be reduced by 93% by reducing the aspect ratio from 8 to 0.125 for constant heat flux boundary condition. While, for constant wall temperature boundary condition, the solidification time reduces by 99% by changing the aspect ratio from 5 to 0.05. In case of convective air boundary surrounding, the solidification time is found to reduce by 88% by reducing the aspect ratio from 8 to 0.125. Based on the analytical solution, correlations have been proposed to predict solidification time in terms of aspect ratio and end-wall boundary condition.

KEYWORDS
constant temperature phase transition, interface of solid and liquid, latent heat thermal energy storage (LHTES), phase change material (PCM), solidification time

Nomenclature:
English Symbols: $T_s$, Temperature of solid PCM, °C; $T_f$, Temperature of fin, °C; $L_p$, PCM’s heat of fusion, J/kg; $T_m$, Solidification temperature of PCM, °C; $c_s$, Solid PCM’s specific heat, J/kg/K; $l_c$, PCM cell’s half height, m; $q_w$, Wall heat flux, W/m²; $t$, Time, s; $X(t)$, Interface of solid and liquid (direction X), m; $Y(t)$, Interface of solid and liquid (direction Y), m; $S_{ts}$, $S_{tl}$, $c_s\Delta T/L_p$, $c_l\Delta T/L_p$ (Stefan’s number); $k$, Thermal conductivity of PCM, W/m°C; $k_f$, Thermal conductivity of fin material, W/m°C; $h$, Coefficient of heat transfer during convection, W/m²/K; $l_f$, Length of fin, m; $H_{ts}h/l_c$, $H_{tf}h/l_f$, $x$, $y$, $r$, Coordinates
Greek Symbols: $\alpha_s$, $\alpha_l$, Solid and liquid PCM’s thermal diffusivity, m²/s; $\alpha_f$, Fin material’s thermal diffusivity, m²/s; $\rho$, PCM’s density, kg/m³; $\delta$, Fin’s half thickness, m; $\sigma$, $l_f/l_c$ (Aspect ratio)
Subfixes: $l$, Liquid; $p$, PCM; $s$, Solid; $f$, Fin
1 | INTRODUCTION

The demand of energy in various sectors such as industrial, utility, and commercial sectors vary with time.1 Thermal energy storage (TES) systems have the ability and are promising candidates for smoothing the temporal variation of energy demands. In addition, TES can be used to utilize the renewable energy sources and improve the energy efficiency. TES can be achieved by using sensible or latent heat storage. The latter is more preferable to the former because of constant temperature phase transition and high storage density.2,3 Phase change materials (PCMs) based Latent heat thermal energy storage (LHTES) for storing thermal energy has various applications such as photovoltaic (PV) system, solar water heating, electronic cooling and thermal comfort. Important advancements in LHTES systems, materials and applications have recently been reviewed by various researchers.4-9 Also, novel PCMs such as pork fat have recently been used by Nizetic et al10 for PV systems.

Heat transfer in LHTES materials, namely, PCMs, is complex in nature because of nonlinear problem and moving solid–liquid interface. The moving solid–liquid problem is commonly termed as moving boundary problem (MBP).1,3 The analytical solution for the one-dimensional (1D) MBP termed as Stefan's problem.11,12 Analytical solutions of 1D MBP for different boundary conditions were reported by Alexiades and Solomon13 and Ozisik.12 These include the perturbation method, the Megerlin method, the quasi-stationary approximation, Kantorovich method, and the heat balance integral method (HBIM).

Several theoretical14-22 and numerical23-26 models were proposed to obtain the PCM temperature variation during solidification and melting. Solomon et al14 reported a solution for rectangular TES with convective air boundary condition by using quasi-stationary approximation. Lu15 presented 1D conduction-based heat transfer model to investigate melting of PCM in Cartesian coordinates. Imposed heat flux was taken on the PCM top surface, while the PCM bottom surface was taken as convective air environment. Authors reported PCM temperature distribution in terms of Biot number and time. Kothari et al17 developed a mathematical model for obtaining the PCM temperature distribution in an annulus by using Variational formulation or quasi-steady approximation. In addition to this, several numerical methods were reported to obtain the solution for moving boundary problem. In general, the enthalpy method is commonly used because of the elimination of explicit treatment of boundary conditions at solid–liquid interface.23 Zivkovich and Fuji24 introduced enthalpy formulation-based mathematical model and simulated the transient behavior of encapsulated PCM in a container. Good agreement was obtained between the results and test data.

Low thermal conductivity is the most significant drawback of PCM. Numerous investigations were reported in literature to enhance the thermal conductivity of PCMs with high-conductivity metal particles, metal foam, and fins.27-30 Fins are widely used to enhance the thermal conductivity of PCMs and heat transfer in LHTES system. Several experimental and numerical studies were reported on heat transfer enhancement of LHTES systems with internal fins.31,32 During the experimental study performed by Velraj et al30 with paraffin wax as PCM inside a cylindrical-vertical shell type heat exchanger, the solidification time was found to be inversely proportional to the number of internal longitudinal fins. Costa et al31 numerically analyzed the thermal performance of LHTES system with and without internal fins using enthalpy-based method. It was reported that the fins significantly enhance the thermal conductivity of PCMs. Also, the authors reported the melt fraction and energy stored of the PCM. The melting and solidification process of PCM storage was investigated through both experimentally and numerically by Lamberg et al.32 The investigations were performed with and without fins, and the heat capacity method was found to be more accurate compared with the enthalpy method.

Several analytical studies were reported for analyzing the PCM melting and solidification time with fins.1-3,33-38 The analytical and numerical solutions for the location of interface of solid and liquid and temperature variation during solidification3 and melting33 of PCM with finned two-dimensional storage were reported for constant end-wall temperature condition. It was reported that geometry of PCM and material properties affect the PCM solidification time. Talati et al2 reported the solution for PCM solidification in two-dimensional PCM storage having internal fins by employing both analytical and numerical techniques for constant heat flux wall condition. Position of interface of solid and liquid and temperature variation of fin in a rectangular-finned PCM storage for convective air environment at the boundaries were predicted by Mosaffa et al.1 Bauer37 reported analytical solution for PCMs solidification in fin geometries using effective medium approach with constant end-wall temperature boundary condition. The model is useful to optimize finned TES. Some of the studies that report the analytical solution for PCM storage having internal fins are summarized in Table 1. This includes the solution for location of interface of solid and liquid and solution for various regions.
### Table 1: Solid–liquid interface locations and solutions for regions 1 and 2 for rectangular phase change material (PCM) storage with internal fins reported by various researchers

<table>
<thead>
<tr>
<th>S. No</th>
<th>Reference</th>
<th>Problem Adapted</th>
<th>Boundary Conditions</th>
<th>Solution for Region 1 (along X Direction)</th>
<th>Solution for Region 2 (along Y Direction)</th>
<th>Validation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Mosaffa et al(^1)</td>
<td>Solidification in a finite rectangular PCM storage with internal fins</td>
<td>Constant imposed convective air environment (T_i(x, t) = \frac{h(T_m - T_w)}{k_e} + hX(t))</td>
<td>(\theta(\xi, t) = \sum_0^{\infty} \left[ \frac{c_m}{N(\lambda_m)} \left( \frac{\cos(\lambda_m\xi) + \phi_m \sin(\lambda_m\xi)}{\lambda_m^2 + \eta^2} \right) \right] + \frac{c_m}{N(\lambda_m)} \theta_m )</td>
<td>Yes (numerical)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>[\theta(\xi, t) = \sum_0^{\infty} \left[ \frac{c_m}{N(\lambda_m)} \left( \frac{\cos(\lambda_m\xi) + \phi_m \sin(\lambda_m\xi)}{\lambda_m^2 + \eta^2} \right) \right] + \frac{c_m}{N(\lambda_m)} \theta_m ]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>Talati et al(^2)</td>
<td>Solidification in a finite rectangular PCM storage with internal fins</td>
<td>Constant imposed end-wall heat flux (T_i(x, t) = -\frac{q_mX(t)}{k_m} + \left[ \frac{\exp(-\alpha_0\rho_0^2)}{(2n-1)^2} \cos(\rho_0x) \right] ) (\psi = \frac{\pi}{\sqrt{2}} \exp\left(\frac{(-\xi) + \psi \left(\frac{1}{2} + \frac{\xi}{2} \right)}{2}\right) + \psi(\eta^2 - \eta)) + \sum_0^{\infty} \left[ \frac{2\lambda^2[(1)^{n+1} - 1]}{\pi \cdot n^2 \pi^2} \exp\left[-(\xi + \sqrt{n^2 \pi^2})^2\right] \right] + \sum_0^{\infty} \left[ \frac{2\lambda^2[(1)^{n+1} - 1]}{\pi \cdot n^2 \pi^2} \cos(n\pi\eta) \right] ) (\theta = -\frac{2\lambda}{\sqrt{\pi}} \exp\left[-(\xi + \sqrt{n^2 \pi^2})^2\right] + \psi(\eta^2 - \eta))</td>
<td>Yes (numerical)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>Lamberg et al(^3)</td>
<td>Solidification in a finite rectangular PCM storage with internal fins</td>
<td>Constant imposed end-wall temperature (S_{m=\infty} = 2\lambda \sqrt{\frac{\alpha_0}{\pi}} \lambda e^{\lambda^2}) (\phi = \frac{\pi}{\sqrt{2}} \exp\left[-(\xi + \sqrt{n^2 \pi^2})^2\right] + \psi(\eta^2 - \eta)) (\theta = \frac{\pi}{\sqrt{2}} \exp\left[-(\xi + \sqrt{n^2 \pi^2})^2\right] + \psi(\eta^2 - \eta)) (\theta = \frac{\pi}{\sqrt{2}} \exp\left[-(\xi + \sqrt{n^2 \pi^2})^2\right] + \psi(\eta^2 - \eta)) (\theta = \frac{\pi}{\sqrt{2}} \exp\left[-(\xi + \sqrt{n^2 \pi^2})^2\right] + \psi(\eta^2 - \eta)) (\theta = \frac{\pi}{\sqrt{2}} \exp\left[-(\xi + \sqrt{n^2 \pi^2})^2\right] + \psi(\eta^2 - \eta)) (\theta = \frac{\pi}{\sqrt{2}} \exp\left[-(\xi + \sqrt{n^2 \pi^2})^2\right] + \psi(\eta^2 - \eta)) (\theta = \frac{\pi}{\sqrt{2}} \exp\left[-(\xi + \sqrt{n^2 \pi^2})^2\right] + \psi(\eta^2 - \eta)) (\theta = \frac{\pi}{\sqrt{2}} \exp\left[-(\xi + \sqrt{n^2 \pi^2})^2\right] + \psi(\eta^2 - \eta)) (\theta = \frac{\pi}{\sqrt{2}} \exp\left[-(\xi + \sqrt{n^2 \pi^2})^2\right] + \psi(\eta^2 - \eta))</td>
<td>Yes (numerical)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td>Lamberg et al(^4)</td>
<td>Melting in a semi-infinite rectangular PCM storage with internal fins</td>
<td>Constant imposed end-wall temperature (S_{i}(t) = 2\lambda \sqrt{\frac{\alpha_0}{\pi}} \lambda e^{\lambda^2}) (\theta = \frac{\pi}{\sqrt{2}} \exp\left[-(\xi + \sqrt{n^2 \pi^2})^2\right] + \psi(\eta^2 - \eta)) (\theta = \frac{\pi}{\sqrt{2}} \exp\left[-(\xi + \sqrt{n^2 \pi^2})^2\right] + \psi(\eta^2 - \eta)) (\theta = \frac{\pi}{\sqrt{2}} \exp\left[-(\xi + \sqrt{n^2 \pi^2})^2\right] + \psi(\eta^2 - \eta)) (\theta = \frac{\pi}{\sqrt{2}} \exp\left[-(\xi + \sqrt{n^2 \pi^2})^2\right] + \psi(\eta^2 - \eta)) (\theta = \frac{\pi}{\sqrt{2}} \exp\left[-(\xi + \sqrt{n^2 \pi^2})^2\right] + \psi(\eta^2 - \eta))</td>
<td>Yes (numerical)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(Continues)
<table>
<thead>
<tr>
<th>S. No</th>
<th>Reference</th>
<th>Problem Adapted</th>
<th>Boundary Conditions</th>
<th>Solution for Region 1 (along X Direction)</th>
<th>Solution for Region 2 (along Y Direction)</th>
<th>Validation</th>
</tr>
</thead>
</table>
| 5.    | Mosaffa et al\textsuperscript{34} | Solidification in a shell and tube PCM storage with internal fins | Constant imposed convective air environment            | \[
T_l - T_m = \ln \left( \frac{L}{a} \right) - \sum_{m=1}^{\infty} \frac{\exp \left( - \alpha_s \gamma_m \right)}{N(y_m)} \left[ Y_0(y_m \Omega) f_1(y_m \Omega f_2) \Omega^2 \left( f_0(\gamma_m \Omega) - J_0(y_m \Omega) f_1(y_m \Omega) \right) \frac{1}{1/(H_e R_m) + \ln \left( \frac{\Omega}{R_m} \right)} \right]
\]
|                                               | where \( J_0 \) and \( Y_0 \) is Bessel function of zero order. | where \( U_0 Y_0(\gamma_m \Omega) - W_0 J_0(\gamma_m \Omega) = 0 \) |

Abbreviation: PCM, phase change material.
In case of TES systems, the melting and solidification time of PCMs affect the thermal performance. In general, the solidification time is nearly three times greater than the melting time.\(^3\) The solidification behavior of PCM is found to depend on various thermal and geometrical parameters including the aspect ratio, fin thickness, and thermophysical properties of fin and PCM.\(^{37}\) It is evident from the literature that numerous studies have been made that analyze the melting/solidification behaviors of PCMs with internal fins inside storage system involving rectangular and cylindrical geometries.\(^{1-3,33-38}\) Most of the two-dimensional analytical studies employ separation of variable (SV) method for the analysis. These techniques compute the solution by truncating the series of complex function. The truncation of infinite series of a function may lead to errors in the solution. Therefore, there is a need to develop theoretical models during solidification of PCM with other methods involving important parameters. HBIM is one of many analytical methods used to solve conduction problems. HBIM has been used to solve Stefan problems because of its simplicity, short computation time while providing the solutions with reasonable accuracy.\(^{39}\)

Here, an effort has been made to obtain the behavior of PCM during solidification process in two-dimensional rectangular PCM storage system with horizontal internal fins subjected to various end-wall boundary conditions, namely, constant heat flux, end-wall temperature, and convective air environment. A simplified 1D model is developed to estimate the location of interface of solid and liquid and temperature variation of fin. Because of symmetry of PCM storage unit, a single symmetry cell was considered for the analysis and is shown in Figure 2. Two regions, namely, Region 1 and Region 2, is considered for the analysis. In Region 1, the transfer of heat is 1D, toward direction X. Here, heat is transferred only by walls, and the fin does not affect the solidification of PCM as reported by Lamberg and Siren.\(^{3,33}\) In Region 2, transfer of heat to the PCM takes place by the wall as well as the fin. After short duration, heat is mainly transferred by the fin. In such case, the interface of solid and liquid is assumed to move one dimensionally in direction Y. In solidification process, conduction is assumed to be primary mode of transfer of heat. The natural convection, at the interface of solid and liquid because of the difference in temperature of liquid PCM, has minimal effect on location of interface of solid and liquid compared with heat conduction in solid PCM.\(^1\) Therefore, the natural convection is not considered in the analysis. The heat transfer problem is complex because of the nonlinear and transient nature. The following assumptions are made for the analysis.\(^1-3\)

(i) Initially, the temperature of PCM and fin is considered to be the same as that of solidification temperature (\(T_m\)).
(ii) The temperature variation of fin is 1D because of small thickness, geometry and high thermal conductivity.
(iii) Various properties of fin, PCM, and heat transfer fluid (HTF) are considered to be constant irrespective of temperature.
(iv) PCM is assumed to be homogeneous, and solidification is considered to take place isothermally.

2 | THE PHYSICAL PROBLEM

Figure 1 represents the schematic of PCM storage unit with internal horizontal fins. Here, left and right walls are subjected to constant temperature, constant heat flux, and constant convective air environment conditions. A simplified 1D model is developed to estimate the location of interface of solid and liquid and temperature variation of fin. Because of symmetry of PCM storage unit, a single symmetry cell was considered for the analysis and is shown in Figure 2. Two regions, namely, Region 1 and Region 2, is considered for the analysis. In Region 1, the transfer of heat is 1D, toward direction X. Here, heat is transferred only by walls, and the fin does

3 | MATHEMATICAL FORMULATION

The present analytical model was developed in two parts. The problem can be treated as Stefan’s problem\(^{13}\) in
Region 1, which is specifically solvable MBP. In Region 2, transfer of heat is assumed to follow the direction Y, and energy balance is required for the analysis. The problem is further divided based on different boundary conditions. This has been elaborated in subsequent sections.

3.1 | Constant imposed heat flux at the walls

3.1.1 | Region 1, \(0 \leq x \leq X(t)\)

The transient heat conduction equation in one dimension for the PCM (Figure 2A) can be expressed as follows:

\[
\frac{\partial^2 T_s}{\partial x^2} = \frac{1}{\alpha_s} \frac{\partial T_s}{\partial t}, \quad 0 \leq x \leq X(t), \quad t = 0.
\]

The initial and boundary conditions are given by:

\[
T_s(x, 0) = T_m, \quad 0 \leq x \leq l_f,
\]

\[
-\rho L_p \frac{dX(t)}{dt} = -k_s \frac{\partial T_s}{\partial x}(X(t), t),
\]

\[
-k_s \frac{\partial T_s}{\partial x} = q^*_{w} \quad q^*_{w} < 0.
\]

Here, \(X(t)\) is the distance of interface of solid and liquid in direction \(X\).

On integrating the governing Equation 1 from end-wall to solid–liquid interface with respect to \(x\) and utilizing the initial and boundary conditions, Equations 2a to 2c and 1 can be rewritten as follows:

\[
\frac{d}{dt} \int_0^{X(t)} \rho c_s [T_s(x, t) - T_m] dx = -\rho L_p X'(t) - q^*_{w}(t), \quad 0 < x < X(t).
\]

In order to solve the above equation and boundary conditions (Equations 2a–2c), a guess temperature profile is assumed and is given as follows:

\[
T_s(x, t) = T_m + A(t)[x - X(t)] + B(t)[x - X(t)]^2,
\]

where, \(A(t)\) and \(B(t)\) are the constants; given by:

\[
A(t) = \frac{L_p}{2c_s X(t)} \left[-1 + \sqrt{1 - \frac{4c_s q^*_{w} X(t)}{k_s L_p}}\right],
\]

\[
B(t) = \frac{L_p}{8c_s \{X(t)\}^2} \left[-1 + \sqrt{1 - \frac{4c_s q^*_{w} X(t)}{k_s L_p}}\right]^2.
\]

Utilizing Equations 4 to 6, polynomial form solution for various parameters such as \(c_s, q^*_{w}, k_s, \rho, L_p, X(t)\), and \(t\). This can be written in the following form.

\[
A_1 \{X(t)\}^4 + B_1 \{X(t)\}^3 + C_1 \{X(t)\}^2 + D_1 X(t) + E_1 = 0
\]

where,

\[
A_1 = [c_s q^*_{w}]^2,
\]
where, $\frac{\partial}{\partial t}q^*_w = \dot{E}_{st}$, 

where, $\dot{E}_{st}$ is the rate of change of energy storage per unit area of the fin. In such case, one can consider the heat transfer takes place because of conjugate effect of heat transfer through fin. Figure 3 shows the arbitrary differential element $dx$, which is detached from the PCM storage. An energy balance for the element $dx$ can be written as follows:

$$\dot{q}_x + \dot{q}_{x+dx} + \dot{q}_{in} = \dot{E}_{st},$$  (14)

where, $\dot{E}_{st}$ is the rate of change of energy storage per unit area of the fin. Substituting rate equations, the energy balance equation (Equation 14) can be rewritten as follows:

$$\frac{\partial^2 T_f}{\partial x^2} - M(T_f - T_m) = \frac{1}{\alpha_f} \frac{\partial T_f}{\partial t},$$  (15)

where,

$$M = \frac{k_f}{k_f \delta Y(t)},$$  (16)

In order to obtain $X(t)$, we need values of various properties of PCM such as $c_s$, $q^*_w$, $k_s$, $\rho$, $L_p$, and $t$. The values of various properties of PCM are summarized in Table 3.

### 3.1.2 Region 2, $\{0 \leq x \leq Y(t)\}$

Here, the interface of solid and liquid moves only in one direction that is in direction $Y$. In such case, one can consider the heat transfer takes place because of conjugate effect of heat transfer through fin. Figure 3 shows the arbitrary differential element $dx$, which is detached from the PCM storage. An energy balance for the element $dx$ can be written as follows:

$$\dot{q}_x + \dot{q}_{x+dx} + \dot{q}_{in} = \dot{E}_{st},$$  (14)

where, $\dot{E}_{st}$ is the rate of change of energy storage per unit area of the fin. Substituting rate equations, the energy balance equation (Equation 14) can be rewritten as follows:

$$\frac{\partial^2 T_f}{\partial x^2} - M(T_f - T_m) = \frac{1}{\alpha_f} \frac{\partial T_f}{\partial t},$$  (15)

where,

$$M = \frac{k_f}{k_f \delta Y(t)},$$  (16)

in which $\delta$ is the half thickness of fin and $Y(t)$ is the distance of interface of solid and liquid in direction $Y$.

The initial and boundary conditions of the fin are given by:

$$T_f(x, 0) = T_m \quad 0 \leq x \leq l_f,$$  (17a)

$$-k_f \frac{\partial T_f(0, t)}{\partial x} = k_f \frac{\partial T_f(l_f, t)}{\partial x} = q^*_w.$$  (17b)

On integrating the governing equation along the half-length of fin with respect to $x$, Equation 15 can be rewritten as follows:

$$\frac{1}{\alpha_f} \frac{d}{dt} \int_0^{l_f/2} (T_f - T_m) \, dx = \int_0^{l_f/2} \left( \frac{\partial T_f(x, t)}{\partial x} \right)_0^{l_f/2} \, dx - M \int_0^{l_f/2} (T_f - T_m) \, dx.$$  (18)

The solution of Equation 15 for different boundary conditions has been reported by several researchers.\textsuperscript{1-3} While, for HBIM method, one needs to assume the guess temperature profile, and this can be written as follows:

$$T_f(x, t) = T_m + t \left[ A(t)e^{\sqrt{M}x} + B(t)e^{-\sqrt{M}x} + C(t) \right].$$  (19)

Applying the initial and boundary conditions given by Equations 17a and 17b of the fin in Equation 18, and substituting in Equation 19, one can obtain the temperature variation of fin and is given by:

$$T_f(x, t) = T_m + t \left[ \frac{q^*_w}{k_f \sqrt{M}} \left( e^{\sqrt{M}x} + e^{-\sqrt{M}x} + e^{\sqrt{M}(x-l_f)} + e^{-\sqrt{M}(x-l_f)} \right) \right] \left( e^{\sqrt{M}l_f} - e^{-\sqrt{M}l_f} \right),$$  (20)

where,

$$P_1 = \frac{2\alpha_f q^*_w}{l_f k_f (1 + M \alpha_f t)} - \frac{Q_1}{l_f \sqrt{M}} \left( e^{\sqrt{M}l_f} - 1 \right) - \frac{R_1}{l_f \sqrt{M}} \left( 1 - e^{-\sqrt{M}l_f} \right),$$  (21a)

$$Q_1 = \frac{q^*_w}{k_f t \sqrt{M}} \left[ \frac{1 + e^{-\sqrt{M}l_f}}{e^{\sqrt{M}l_f} - e^{-\sqrt{M}l_f}} \right],$$  (21b)

$$R_1 = \frac{q^*_w}{k_f t \sqrt{M}} \left[ \frac{1 + e^{\sqrt{M}l_f}}{e^{\sqrt{M}l_f} - e^{-\sqrt{M}l_f}} \right].$$  (21c)
The location of interface of solid and liquid in Region 2, that is in direction Y, Y (t) is defined following the Megerlin method as below.

\[ Y(t) = 2 \left[ \frac{\sqrt{1 + 2S_{te} - 1}}{2} \right]^{1/2} \sqrt{\alpha_s t} \] Megerlin method, (22)

where, \( S_{te} \) denotes the Stefan number and is provided by:

\[ S_{te} = \frac{c_s(T_m - T_f)}{L_p}. \] (23)

It may be noted by using Equations 20 to 23, one can obtain temperature variation of the fin and the location of interface of solid and liquid Y (t).

### 3.2 | Constant imposed temperature at the walls

Figure 2B shows the location of interface of solid and liquid during solidification in two different regions for constant imposed temperature at the end-walls.

#### 3.2.1 | Region 1, \( 0 \leq x \leq X(t) \)

Here, the solution of the model is obtained by considering similar governing equation (Equation 1), initial condition (Equations 2a and 2b), and assumed temperature profile (Equation 4).

In addition to this, boundary condition for constant imposed end-wall heat flux (Equation 2c) is replaced with constant end-wall temperature boundary condition and is given as follows:

\[ T_s(0, t) = T_s(l_f, t) = T_w < T_m. \] (24)

On integrating the governing equation (Equation 1) from end wall to interface of solid and liquid with respect to x and utilizing the initial and boundary conditions given by Equations 2a, 2b, 24, and 1 can be rewritten as follows:

\[
\frac{d}{dt} \int_0^X dx \rho c_s [T_f(x, t) - T_m] = -\rho L_p X'(t) - k_s T_s(0, t) \quad \text{for} \quad 0 < x < X(t).
\] (25)

\( A(t) \) and \( B(t) \) are the constants in assumed temperature profile, and their values are obtained as follows:

\[ A(t) = \frac{L_p}{c_s X(t)} [ -1 + \sqrt{1 - 2S_{te}} ], \] (26)

\[ B(t) = \frac{L_p}{2c_s X(t)} [ -1 + \sqrt{1 - 2S_{te}} ]^2. \] (27)

Utilizing Equations 4 and 25 to 27, the position of interface of solid and liquid is given as follows:

\[ X(t) = 2 \left[ \frac{3(-1 + 2S_{te} - \sqrt{1 - 2S_{te}})}{-7 - 2S_{te} + \sqrt{1 - 2S_{te}}} \right] \sqrt{\alpha_s t}. \] (28)

#### 3.2.2 | Region 2, \( 0 \leq x \leq Y(t) \)

It may be noted that solution of Region 2 for this model is obtained by considering similar governing equation (Equation 15), initial condition (Equation 17a) and assumed temperature profile (Equation 19).

Apart from above equations, following boundary conditions for constant imposed end-wall temperature boundary condition and is given as follows:

\[ T_f(0, t) = T_f(l_f, t) = T_w. \] (29)

Applying the initial and boundary conditions of the fin (Equations 17a and 29) in Equation 18 and substituting in Equation 19, one can obtain the temperature variation of fin and is given by:

\[ T_f(x, t) = T_w + tP_2 \left[ e^{\sqrt{M} \sqrt{X-l_f}} + e^{-\sqrt{M} \sqrt{X}} - \left( 1 + e^{-\sqrt{M} l_f} \right) \right], \] (30)

where,

\[
P_2 = \frac{(T_m - T_w) l_f \sqrt{M}}{2t \left[ e^{-\sqrt{M} l_f} \left( e^{(\sqrt{M} l_f)/2} - 1 \right) \right] + \left( 1 - e^{-\sqrt{M} l_f} \right)} - \frac{l_f \sqrt{M}}{2} \left( 1 + e^{-\sqrt{M} l_f} \right) + \frac{M \alpha_s t \left( e^{-\sqrt{M} l_f} - 1 \right)}{1 + M \alpha_s t}.
\] (31a)
Here, the interface of solid and liquid $Y(t)$ is defined similar to the Equation 22. Also, variation in temperature of the fin and the location of interface of solid and liquid $Y(t)$ can be obtained using Equations 22 23 and 30-(31a).

### 3.3 Convective air environment at the walls

The location of interface of solid and liquid during solidification in two different regions for convective air environment boundary condition is shown in Figure 2C.

#### 3.3.1 Region 1, $0 \leq x \leq X(t)$

In addition to governing Equation 1, initial and boundary conditions (Equations 2a-2b) and assumed temperature profile (Equation 4), constant air environment boundary condition is used instead of constant imposed end-wall heat flux (Equation 2c) to obtain the solution. And the solution is given as follows:

$$\frac{-\partial T_s(0,t)}{\partial x} + H_s\{T_s(0,t) - T_\infty\} = 0. \tag{34}$$

The governing Equation 1 can be rewritten by integrating it with respect to $x$ from end wall to interface of solid and liquid, and substituting the initial and boundary conditions (Equations 2a, 2b, and 34), we can obtain the following relation:

$$\frac{d}{dt} \int_0^{X(t)} \rho c_s[T_s(x,t) - T_m]dx = -\rho L_pX'(t)$$

$$\int_0^{X(t)} \frac{-h[T_\infty - T(0,t)]}{\partial x} \, dx \text{ for } 0 < x < X(t). \tag{35}$$

Assuming,

$$\frac{k_s + hX(t)}{2k_s + hX(t)} \approx 1, \tag{36a}$$

$$\frac{2hX(t)}{k_s + hX(t)} \approx 1. \tag{36b}$$

The value of constants $A(t)$ and $B(t)$ assumed in temperature profile (Equation 4) are obtained as follows:

$$A(t) = \frac{L_p}{c_sX(t)}\left[\sqrt{1 + st_s} - 1\right], \tag{37}$$

$$B(t) = \frac{-L_p}{2c_s\{X(t)\}^2}\left[\sqrt{1 + st_s} - 1\right]^2, \tag{38}$$

where, $st_s$ is Stefan number for solid PCM and is given by $st_s = c_s(T_m - T_\infty)$.

Utilizing Equations 4, 35, 37, and 38, the solid–liquid interface position is given as follows:

$$X(t) = \frac{3ht}{\rho c_s}\left[st_s - 2\{1 - \sqrt{1 + st_s}\}\right]. \tag{39}$$

#### 3.3.2 Region 2, $0 \leq x \leq Y(t)$

In addition to Governing Equation 15, initial condition (Equation 17a) and assumed temperature profile (Equation 19), constant air environment boundary condition is used instead of constant imposed end-wall heat flux (Equation 17b) for obtaining the solution. The solution is given as follows:

$$\frac{-\partial T_f(0,t)}{\partial x} + H_f\{T_f(0,t) - T_\infty\}$$

$$= \frac{\partial T_f(0,t)}{\partial x} + H_f\{T_f(0,t) - T_\infty\} = 0. \tag{40}$$

The temperature variation of fin is obtained by substituting Equations 17a, 18, and 40 in Equation 19 given as follows:

$$T_f(x,t) = T_\infty + tP_3\left[Q_3e^{\sqrt{M}x} + e^{-\sqrt{M}x}\right]$$

$$+ \left(Q_3 \times \left(\frac{\sqrt{M}}{H_f} - 1\right) - 1 - \frac{\sqrt{M}}{H_f}\right), \tag{41}$$

where,

$$P_3 = \left\{\frac{(T_m - T_\infty)l_f}{t}\right\}$$

$$\times \left\{R_1\left(\frac{\sqrt{M}}{H_f} - 1\right) - 1 - \frac{\sqrt{M}}{H_f}\right\}\left(l_f + 2S_3\right) + \left[R_3 + e^{-\sqrt{M}l_f}\right]$$

$$\times \left\{\left(\frac{\sqrt{M}l_f}{M} - 1\right) + S_3 \left(1 + e^{\sqrt{M}l_f}\right)\right\}. \tag{42a}$$
Here, the interface of solid and liquid \( Y(t) \) is defined similar to the Equation 22. Also, temperature variation of fin and location of interface of solid and liquid \( Y(t) \) can be obtained using the Equations 22, 23, and 41 to 42d.

\[ Q_3 = \frac{\sqrt{M} e^{-\sqrt{M} \hat{l}_j}}{H_f} + \frac{\sqrt{M} \hat{l}_j}{H_f} + 1 - e^{-\sqrt{M} \hat{l}_j} \]  
\[ R_3 = \frac{\alpha_f H_f t}{1 + M \alpha_f t} \]  
\[ M = \frac{k_s}{k_f 5Y(t)} \]  

4 | COMPARISON OF PRESENT PREDICTION WITH NUMERICAL RESULTS

In order to verify the present analytical model, efforts have been made to compare the location of interface of solid and liquid and solid fraction of PCM obtained from the present prediction with the two-dimensional numerical solution of Talati et al., Lamberg et al., and Mosaffa et al. for constant heat flux, end-wall temperature, and convective air boundary condition, respectively, and the results of the validation are shown in Figure 4A-C and Table 2A-C. For the sake of brevity, the location of solid and liquid interface in directions X and Y are compared at \( \sigma < 1 \). The average error, between the present predictions and two-dimensional numerical results, in obtaining the location of interface of solid and liquid in direction X is found to be 0.69, 1.73, and 1.06 mm for constant heat flux, end-wall temperature, and convective air boundary condition, respectively. While, the average error in obtaining the location of interface of solid and liquid in direction Y is found to be 0.19, 0.39, and 0.57 mm in direction Y for constant heat flux, end-wall temperature, and convective air boundary condition, respectively.

The solid fraction of PCM during solidification for varied range of aspect ratio and different boundary condition are compared with the two-dimensional numerical results.
solutions and is summarized in Table 2A. Present prediction exhibits good agreement with the numerical solution. The maximum error between the present prediction and the two-dimensional numerical solution is found to be \(\pm 13\%\). Therefore, the model can be useful in designing the PCM-based TES systems.

**TABLE 2A** Comparison of present analytical model and two-dimensional numerical model\(^2\) for constant heat flux boundary condition in different test cases

<table>
<thead>
<tr>
<th>Aspect Ratio ((\sigma))</th>
<th>Time ((s))</th>
<th>Solid Fraction Predicted from Present Study ((\epsilon_{\text{anal}}))</th>
<th>Solid Fraction Obtained from Two-dimensional Numerical Model,(^2) ((\epsilon_{\text{num}}))</th>
<th>Error ((I-II)\times 100)%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>400</td>
<td>0.43</td>
<td>0.35</td>
<td>6.7</td>
</tr>
<tr>
<td>1</td>
<td>400</td>
<td>0.30</td>
<td>0.25</td>
<td>3.3</td>
</tr>
<tr>
<td>2</td>
<td>400</td>
<td>0.26</td>
<td>0.21</td>
<td>1.4</td>
</tr>
</tbody>
</table>

**TABLE 2B** Comparison of present analytical model and two-dimensional numerical model\(^3\) for constant temperature boundary condition in different test cases

<table>
<thead>
<tr>
<th>Aspect Ratio ((\sigma))</th>
<th>Time ((s))</th>
<th>Solid Fraction Predicted from Present Study ((\epsilon_{\text{anal}}))</th>
<th>Solid Fraction Obtained from Two-dimensional Numerical Model,(^3) ((\epsilon_{\text{num}}))</th>
<th>Error ((I-II)\times 100)%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paraffin, 0.2</td>
<td>724</td>
<td>0.90</td>
<td>0.79</td>
<td>10.6</td>
</tr>
<tr>
<td>Paraffin, 1</td>
<td>1085</td>
<td>0.29</td>
<td>0.23</td>
<td>6.1</td>
</tr>
<tr>
<td>Paraffin, 5</td>
<td>1085</td>
<td>0.61</td>
<td>0.53</td>
<td>7.1</td>
</tr>
</tbody>
</table>

**TABLE 2C** Comparison of present analytical model and two-dimensional numerical model\(^1\) for convective air boundary condition in different test cases

<table>
<thead>
<tr>
<th>Aspect Ratio ((\sigma))</th>
<th>Time ((s))</th>
<th>Solid Fraction Predicted from Present Study ((\epsilon_{\text{anal}}))</th>
<th>Solid Fraction Obtained from Two-dimensional Numerical Model,(^1) ((\epsilon_{\text{num}}))</th>
<th>Error ((I-II)\times 100)%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>350</td>
<td>0.27</td>
<td>0.40</td>
<td>-13.0</td>
</tr>
<tr>
<td>1</td>
<td>500</td>
<td>0.27</td>
<td>0.36</td>
<td>-9.4</td>
</tr>
<tr>
<td>2</td>
<td>500</td>
<td>0.20</td>
<td>0.23</td>
<td>-3.0</td>
</tr>
</tbody>
</table>

5 | RESULTS AND DISCUSSION

Here, an analytical model was developed to analyze the temperature distribution in both PCM and fin during the solidification of PCM inside the rectangular container with horizontal internal fins. The fraction of the fin length to the half-height of symmetry section \((l_f/l_c)\) is considered as the cell aspect ratio \((\sigma)\) (see Figure 1). Various cases with different values of \(\sigma\) are considered for the analysis. For all the cases, half of the thickness of fin is considered to be 0.5 mm. Initially, the liquid PCM is at the solidification temperature \((T_m)\) inside the PCM storage unit. HBIM was employed to solve the conduction equation associated with different boundary conditions. Different guess profiles for temperature distribution were used for the analysis. Furthermore, the present predictions were compared with the results obtained by existing analytical and numerical techniques.

5.1 | Constant imposed heat flux at the wall

The PCM and fin material considered for the present study are salt hydrate Climsel C28 and aluminum, respectively. Their properties are summarized in Table 3. The different values of \(\sigma\) are taken as 0.5, 1, and 2 for the analysis. Dimensions of PCM storage unit are shown in Table 4. Constant heat flux, \(q''_w = -1000\ \text{W/m}^2\), was taken for solidification of PCM in all the cases.

**TABLE 3** Thermal properties of phase change material and fin material\(^2\)

<table>
<thead>
<tr>
<th>Property</th>
<th>Paraffin</th>
<th>Aluminum Fin</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density, (\rho) (kg/m(^3))</td>
<td>1450</td>
<td>2770</td>
</tr>
<tr>
<td>Specific heat, (c) (J/kg·K)</td>
<td>3600</td>
<td>875</td>
</tr>
<tr>
<td>Thermal conductivity, (k) (W/m·K)</td>
<td>0.6</td>
<td>177</td>
</tr>
<tr>
<td>Latent heat of fusion, (L_p) (J/kg)</td>
<td>162 000</td>
<td>...</td>
</tr>
<tr>
<td>Melting/solidification temperature, (T_m) (°C)</td>
<td>28</td>
<td>...</td>
</tr>
</tbody>
</table>

**TABLE 4** Dimensions of phase change material storage unit

<table>
<thead>
<tr>
<th>Dimensions</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of fin, (l_f) (mm)</td>
<td>10</td>
<td>15</td>
<td>20</td>
</tr>
<tr>
<td>Half height of cell, (l_c) (mm)</td>
<td>20</td>
<td>15</td>
<td>10</td>
</tr>
<tr>
<td>Cell aspect ratio, (\sigma = \frac{l_f}{l_c})</td>
<td>0.5</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>
dimensional numerical solutions of Talati et al\textsuperscript{2} for different values of time steps (t) and $\sigma$. Present predictions exhibit good agreement with the 1D results obtained by employing SV method and two-dimensional results obtained by finite difference method.\textsuperscript{2}

It is evident that geometry of PCM storage changes the solidification rate. When $\sigma$ is very less than unity, heat flows only from wall to interface of solid and liquid, in direction X. When $\sigma$ is much larger than unity, heat flows only from fin to interface of solid and liquid, in direction Y. In case of $\sigma$ equal to unity, heat flows from both wall and fin to interface of solid and liquid, in the directions X and Y. Figure 5 shows the comparison of results obtained by using integral method in the present study with various available mathematical models. Here, the results of 1D analytical and two-dimensional numerical models of Talati et al\textsuperscript{2} are considered for comparison. In all the cases, sharp corners are formed on the interface of solid and liquid at the point near both the wall and the fin surfaces. This is because of the high temperature at the corners. The maximum error in solid–liquid interface between the two sets of results was found to be 0.96 mm in direction X and 0.40 mm in direction Y at $\sigma = 2$ and $t = 400$ seconds. However, present model exhibits
excellent agreement with the two-dimensional numerical model when the value of $\sigma = 0.5$.

The temperature distributions of fin obtained from the present analytical model are compared with the results obtained by employing SV and finite difference methods, and are shown in Figures 6A-C. It is observed from Figure 6A that the present predictions exhibit good agreement with the results obtained by SV method. The maximum error in temperature difference between both sets of results was found to be very negligible (~0.095°C) at $\sigma = 2$ and $t = 800$ seconds. It has been reported that 1D heat transfer occurs in case of short fins when the value of $\sigma$ is much smaller than unity. The temperature variation along the fin length is found to be low because of high thermal conductivity and constant heat flux. However, the temperature variation increases as $l_f$ increases. The current results exhibit excellent agreement with the numerical solutions obtained by Talati et al. Also, the present predictions agree well with the results obtained by employing SV method.

Solid fraction of PCM usually denotes the amount of solidified PCM in the symmetric storage cell for a given duration. The solid fraction of PCM changes from entirely solid ($\varepsilon = \text{unity}$) to completely liquid ($\varepsilon = 0$) and is expressed as follows:

$$
\varepsilon = \frac{2X(t)[l_c - \delta - Y(t)] + l_f Y(t)}{[l_c - \delta]l_f}.
$$

Time of PCM solidification in a rectangular storage having fins for each symmetry cell was obtained using the analytical formulation for different cases and is presented in Figure 7. It is noticed from Figure 7 that the PCM solidification is fast for smaller values of $\sigma$. Initially, the PCM solidification is faster because of small thickness of solid PCM layer. However, as the time progresses, the thickness of solid PCM layer increases, which increases the thermal resistance and solidification rate decreases.

Figure 8 shows variation in PCM’s total solidification time for different aspect ratio at constant volume of PCM and TCE. It may be noted from the figure that solidification time is less for lower aspect ratio of TES system for constant volume of PCM and TCE. This may be due to the fact that with the decrease in aspect ratio, distance between fins increases and a huge amount of heat is liberated from the end walls. The PCM’s total solidification time is found to be less for lower aspect ratio ($\sigma = 0.125$) compared with $\sigma = 8$ (Figure 8). The reduction in solidification time was found to be approximately 93% by reducing the aspect ratio from 8 to 0.125 for a given imposed heat flux at the walls. In addition to this, with the increase in the imposed heat flux from 500 to

![Figure 6](https://wileyonlinelibrary.com)
2000 W/m², the solidification time reduces by approximately 60% for the same aspect ratio.

In the present study, a correlation has been proposed for solidification time \( t_s \) in terms of aspect ratio and end-wall boundary condition and expressed in Equation 44.

\[
t_s = 125077.867(\sigma)^{0.6788}(q_w^{*})^{-0.5976}. \tag{44}
\]

The proposed correlation for \( t_s \) is valid for the range of \( 0.125 \leq \sigma \leq 8 \) and the error band to anticipate 99% test data of the proposed correlation for solidification time \( t_s \) is found to be ±5% as shown in Figure 9.

5.2 | Constant imposed temperature at the walls

In this study, pure n-octadecane paraffin is used as the PCM with aluminum as fins and commercial grade salt hydrate (Climsel 23) used as the PCM with steel as fins for the analysis. Table 5 summarizes the properties of fin materials and PCM. Different values of \( \sigma \) such as 0.2, 1, and 5 are considered for the analysis. Table 6 shows the dimensions of PCM storage unit for which the constant wall temperature was considered (Figure 2).

It is observed from Figure 10 that the aspect ratio \( \sigma \) of PCM storage unit affects the rate of solidification. The heat flows only in direction X, ie, from wall to interface of
solid and liquid when the value of $$\sigma$$ is less than unity. However, the heat flows only in direction Y, ie, from wall to interface of solid and liquid when the reverse is true. In case of $$\sigma$$ equals to unity, heat flows from both wall to solid–liquid interface and fin to solid–liquid interface, ie, both in X and Y directions. Figures 10A–E depict the comparison of locations of interface of solid and liquid obtained from the present 1D analytical predictions and two–dimensional numerical solutions of Lamberg et al for different values of $$t$$ and $$\sigma$$. The maximum error in predicting interface of solid and liquid by present model was found to be 1.82 and 0.37 mm in X and Y directions, respectively, at $$\sigma = 5$$ and $$t = 1085$$ seconds for n–octadecane. While, the maximum error for salt hydrate was found to be $-1.389^\circ C$ at $$\sigma = 1$$ and $$t = 4226$$ seconds. The present results agree well with the existing numerical results.

The variation of solid fraction with time for two different PCMs for $$\sigma = 0.2$$ and $$\sigma = 5$$ was evaluated using Equation 43 and is demonstrated in Figure 12. It may be noted that for the same aspect ratio the solidification rate is higher for salt hydrate as compared with paraffin because of better heat conductivity of salt hydrate compared with paraffin. This may be due to the difference in density and latent heat of fusion of two PCMs. The reduction in respective solidification rate is found to be 73.4% and 50% for $$\sigma = 0.2$$ and $$\sigma = 5$$ for paraffin and salt hydrate.

Total solidification time of PCM for solidification time of paraffin for different aspect ratio at constant volume of PCM and TCE is presented in Figure 13. The solidification time is found to be less for lower aspect ratio of TES system. This may be because the distance between fins increases with the decrease in aspect ratio and a large amount of heat is discharged from the end walls. The PCM’s total solidification time is remarkably less for lower value of aspect ratio ($$\sigma = 0.05$$) compared with $$\sigma = 5$$ (Figure 13). The reduction in solidification time was found to be approximately 99% by reducing the aspect ratio from 5 to 0.05 for same imposed heat flux at the walls. In addition to this, with the decrease in the imposed end wall from 18 to 8$^\circ C$, the solidification time reduces by approximately 54% for the same aspect ratio.

### Table 5
Thermal properties of phase change materials and fin materials

<table>
<thead>
<tr>
<th>Property</th>
<th>Paraffin n-Octadecane</th>
<th>Salt Hydrate Climson23</th>
<th>Aluminum Fin</th>
<th>Steel Fin</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density, $$\rho$$ (kg/m$^3$)</td>
<td>777</td>
<td>1480</td>
<td>2713</td>
<td>7854</td>
</tr>
<tr>
<td>Specific heat, $$c$$ (J/kg·K)</td>
<td>2660</td>
<td>2660</td>
<td>960</td>
<td>434</td>
</tr>
<tr>
<td>Thermal conductivity, $$k$$ (W/m·K)</td>
<td>0.149</td>
<td>0.6</td>
<td>180</td>
<td>60.5</td>
</tr>
<tr>
<td>Latent heat of fusion, $$L_p$$ (J/kg)</td>
<td>241 360</td>
<td>148 000</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>Melting/solidification temperature, $$T_m$$ (°C)</td>
<td>28</td>
<td>23</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>Viscosity, $$\mu$$ (kg/m·s)</td>
<td>0.00385</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>Thermal expansion coefficient, $$\alpha$$ (K$^{-1}$)</td>
<td>0.001</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

### Table 6
Dimensions of phase change material storage, end-wall temperature, and materials used in different test cases

<table>
<thead>
<tr>
<th>Cases</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
<th>Case 5</th>
<th>Case 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>PCM material</td>
<td>Paraffin</td>
<td>Paraffin</td>
<td>Paraffin</td>
<td>Salt hydrate</td>
<td>Salt hydrate</td>
<td>Salt hydrate</td>
</tr>
<tr>
<td>Fin material</td>
<td>Aluminum</td>
<td>Aluminum</td>
<td>Aluminum</td>
<td>Steel</td>
<td>Steel</td>
<td>Steel</td>
</tr>
<tr>
<td>End-wall temperature, $$T_w$$, (°C)</td>
<td>13</td>
<td>13</td>
<td>13</td>
<td>8</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>Length of fin, $$l_f$$ (mm)</td>
<td>10</td>
<td>50</td>
<td>50</td>
<td>10</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>Half height of cell, $$l_c$$ (mm)</td>
<td>50</td>
<td>50</td>
<td>10</td>
<td>50</td>
<td>50</td>
<td>10</td>
</tr>
<tr>
<td>Cell aspect ratio, $$\sigma = \frac{l_f}{l_c}$$</td>
<td>0.2</td>
<td>1</td>
<td>5</td>
<td>0.2</td>
<td>1</td>
<td>5</td>
</tr>
</tbody>
</table>
In the present investigation, a correlation has been proposed for solidification time \( t_s \) in terms of aspect ratio and end-wall boundary condition and expressed in Equation 46.

\[
t_s = 329.3757(\sigma)^{1.0021}(T_w)^{1.0512}.
\]  

(46)

The proposed correlation for \( t_s \) is valid for the range of \( 0.05 \leq \sigma \leq 8 \) and the error band to anticipate 99% test data of the proposed correlation for solidification time \( (t_s) \) is found to be ±10% as shown in Figure 14.

5.3 | Convective air environment boundary conditions

In this section, paraffin and aluminum are taken as PCM and fin material, respectively. Their properties are summarized in Table 7. Here, different values of \( \sigma \) such as 0.5, 1, and 2 are considered. The dimensions of PCM storage unit are listed in Table 8. Constant air boundary condition was considered for PCM storage unit (Figure 2). HTF temperature and coefficient of heat transfer during convection for PCM solidification were assumed constant in all the cases (Figure 2C). Air was chosen as HTF
having the temperature of 10°C and the value of coefficient of heat transfer during convection at the wall was taken as 65 W/m²/K.¹

Rate of solidification is influenced by the aspect ratio of PCM storage unit. It can be seen from Figure 15 that when \( \sigma \) is less than unity, heat flows only in direction X, that is, from wall to solid–liquid interface. However, heat flows only in direction Y, that is, from fin to interface of solid and liquid, when the value of \( \sigma \) is greater than unity. While, heat flows from both wall to solid–liquid interface and fin to interface of solid–liquid (both in the directions X and Y) at \( \sigma = 1 \). The comparison of location of interface of solid and liquid obtained from the present analytical investigation with the 1D mathematical and two-dimensional numerical solutions of Mosaffa et al.¹ for different values of \( t \) and \( \sigma \) is presented in Figure 15A-F. The maximum error in interface of solid–liquid by the present model was found to be 1.1477 mm in direction X at \( \sigma = 1 \) and \( t = 1000 \) seconds and 0.973 mm in direction Y at \( \sigma = 2 \) and \( t = 1000 \) seconds. The present model agrees well with the two-dimensional numerical model for \( \sigma = 0.5 \).
Here, the fin’s temperature variation was determined and compared with the results obtained by employing SV and enthalpy method,\textsuperscript{1} and is presented in Figure 15 A-C. It is noticed from Figure 16 that the present predictions exhibit good agreement with the result obtained by SV method. The maximum error in temperature difference by the present model was found to be negligible at $\sigma = 2$ and $t = 1000$ seconds. It may also be observed that the length of fin significantly changes the temperature variation. For the small values of $l_f$, the temperature difference is small, while it increases with the values of $l_f$. This is attributed to the 1D heat transfer in the small length of a fin and its higher thermal conductivity. The present predictions agree well with the existing numerical solutions.\textsuperscript{1}

**Figure 12** (A) Variation of solid fraction of phase change material (PCM) with time, (B) solidification time of PCM for different cell aspect ratios and PCMs [Colour figure can be viewed at wileyonlinelibrary.com]

**Figure 13** Variation in total solidification time of phase change material for different imposed temperature at the end walls [Colour figure can be viewed at wileyonlinelibrary.com]

**Figure 14** Comparison of computed solidification temperature with those predicted by the proposed correlation [Colour figure can be viewed at wileyonlinelibrary.com]

**Table 7** Thermal properties of phase change material and fin material\textsuperscript{1}

<table>
<thead>
<tr>
<th>Property</th>
<th>Paraffin</th>
<th>Aluminum Fin</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density, $\rho$ (kg/m$^3$)</td>
<td>830</td>
<td>2770</td>
</tr>
<tr>
<td>Specific heat, $c$ (J/kg·K)</td>
<td>1920 (solid)</td>
<td>3260 (liquid)</td>
</tr>
<tr>
<td>Thermal conductivity, $k$ (W/m·K)</td>
<td>0.514 (solid)</td>
<td>0.224 (liquid)</td>
</tr>
<tr>
<td>Latent heat of fusion, $L_p$ (J/kg)</td>
<td>251 000</td>
<td>...</td>
</tr>
<tr>
<td>Melting/solidification temperature, $T_m$ (°C)</td>
<td>32</td>
<td>...</td>
</tr>
</tbody>
</table>
The variation of solid fraction with time for \( \sigma = 0.5 \) and \( \sigma = 2 \) was evaluated using Equation 43 and is demonstrated in Figure 17. It is noticeable from Figure 17 that the PCM solidification is fast for smaller values of \( \sigma \).

**Table 8** Dimensions of phase change material storage unit

<table>
<thead>
<tr>
<th>Dimensions</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of fin, ( l_f ) (mm)</td>
<td>10</td>
<td>15</td>
<td>20</td>
</tr>
<tr>
<td>Half height of cell, ( l_c ) (mm)</td>
<td>20</td>
<td>15</td>
<td>10</td>
</tr>
<tr>
<td>Cell aspect ratio, ( \sigma = \frac{l_f}{l_c} )</td>
<td>0.5</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

Figure 18 shows variation in PCM total solidification time for different aspect ratio at constant volume of PCM and TCE. Different values of temperature of HTF and coefficient of heat transfer during convection has been taken for the analysis. Here, the reduction in solidification time was found to be approximately 88% by reducing the aspect ratio from 8 to 0.125 for a given imposed convective environment at the walls. Also, with the increase in the coefficient of heat transfer from 55 to 75 W/m²/K, the solidification time reduces by approximately 26% at \( T_\infty = 10°C \) for the same aspect ratio. However, the reduction in solidification time is obtained as approximately
55% by reducing $T_\infty$ from 20 to 10°C at $h = 65$ W/m²/K for the same aspect ratio. Here, it may be noted the decrease of HTF temperature ($T_\infty$) has more effect on solidification time compared with increase in coefficient of heat transfer during convection ($h$).

The proposed correlation for solidification time ($t_s$) in terms of aspect ratio and end-wall boundary condition is expressed in Equation 47.

$$t_s = 125077.867(\sigma)^{0.5014}(h)^{-0.5814}(T_m - T_\infty)^{-1.0002}.$$  \hspace{1cm} (47)

The proposed correlation for $t_s$ is valid for the range of $0.125 \leq \sigma \leq 8$ and $T_m > T_\infty$. The error band to anticipate 99% test data of the proposed correlation for solidification time ($t_s$) is found to be ±10% as shown in Figure 19.
6 | CONCLUSION

A theoretical model has been proposed to estimate various parameters such as solid fraction, interface of solid and liquid, solidification time, and temperature variation during the solidification of PCM in a two-dimensional LHTES with horizontal internal fins. Various boundary conditions, namely, constant heat flux, end-wall temperature, and convective air boundary conditions, were considered at the walls of the storage system. HBIM was used to obtain the solution. The conclusions obtained from the present study are elaborated below.

- Closed form expressions were obtained for various parameters such as temperature distribution, solid-liquid interface, and solid fraction.
- Present predictions agree well with the results obtained by employing SV and finite difference methods.
- The solidification time is found to reduce by 93% and 99% by reducing the aspect ratio from 8 to 0.125 for constant heat flux boundary condition and reducing the aspect ratio from 5 to 0.2 for constant wall temperature boundary condition, respectively.
- For the same aspect ratio, the solidification time is found to be more for salt hydrate compared with paraffin. By reducing the aspect ratio from $\sigma = 0.2$ to $\sigma = 5$, the solidification time was found to reduce by 50% and 73% for paraffin and salt hydrate, respectively. The solidification time was found to decrease by 88% for paraffin by reducing the aspect ratio from 8 to 0.125 for constant convective air boundary condition.
- Correlations for the solidification time is given by

$$t_s = 125077.867(\sigma)^{0.0514}(h)^{-0.5814}(T_m - T_\infty)^{-1.0002}$$

for constant heat flux, end-wall temperature, and convective boundary conditions, respectively. The error band to anticipate 99% test data of the proposed correlation for solidification time ($t_s$) is found to be ±10%.

The present analytical model can contribute to better understanding of solidification of PCM in a finite storage with internal fins. The present study reports the effect of various geometrical and thermal boundary conditions on solidification time. The present analytical model can be useful in optimum design of PCM-based TES system with internal fins.

ACKNOWLEDGEMENT

The financial support received from DST, Government of India (DST Sanction No.: DST/TMD/MES/2k17/65) is gratefully acknowledged by the authors. Also, the financial support received from DST-INSPIRE Fellowship program (IF170534) is admired by the first author.

ORCID

Rohit Kothari [https://orcid.org/0000-0002-7678-6690](https://orcid.org/0000-0002-7678-6690)

REFERENCES

1. Mosaffa AH, Talati F, Rosen MA, Tabirizi HB. Approximate analytical model for PCM solidification in a rectangular finned


**How to cite this article**: Kothari R, Das S, Sahu SK, Kundalwal SI. Analysis of solidification in a finite PCM storage with internal fins by employing heat balance integral method. *Int J Energy Res*. 2019;1-23. [https://doi.org/10.1002/er.4363](https://doi.org/10.1002/er.4363)